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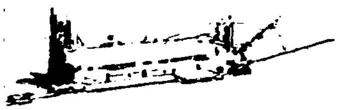

DREDGING RESEARCH PROGRAM

CONTRACT REPORT DRP-93-2

RATIONAL TECHNIQUES FOR EVALUATING THE POTENTIAL OF SANDS FOR BEACH NOURISHMENT

by

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Final Report

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Prepared for U.S. Army Corps of Engineers
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Under Work Unit 32463

The Dredging Research Program (DRP) is a seven-year program of the US Army Corps of Engineers. DRP research is managed in these five technical areas:

- Area 1 - Analysis of Dredged Material Placed in Open Water
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- Area 4 - Vessel Positioning, Survey Controls, and Dredge Monitoring Systems
- Area 5 - Management of Dredging Projects

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Dredging Research Program Report Summary

Rational Techniques for Evaluating Potential Sands for Beach Nourishment (CR DRP-93-2)

ISSUE: The rational design of beach-nourishment projects requires the ability to calculate the final configuration of the added sand volume. This capability is essential for quantitative evaluation of the relative merits of various borrow areas and in benefit/cost analysis of such projects.

RESEARCH: A methodology was developed for predicting the equilibrium beach profile resulting from placement of an arbitrary volume of material with an arbitrary grain-size distribution on a profile of arbitrary shape and grain-size distribution. The methodology depends on the theory of equilibrium profile shape and is proposed as an alternative to traditional compatibility and overfill ratio factors for borrow and native material.

SUMMARY: Various available methods for relating the overall qualities of borrow and native sediments were reviewed. These methods generally focus on comparing grain-size characteristics rather than response to a wave and tide regime. Thus the procedures must be considered ad hoc and the results not truly representative of a beach nourishment material. Also, the methods cannot be used to determine additional dry beach width, a factor necessary in benefit/cost analysis and for project design and planning operations.

A methodology and a computer program were developed for predicting beach shapes relevant to beach nourishment using sediments of arbitrary sorting. The theory was then applied in four specific examples covering a range of beach, native sediment, and fill material conditions. Two sets of field data were located that provided comparison with and evaluation of the general methodology.

The method presented in the report is considered by the authors to be one step toward a rational procedure for assessing the complex performance of nourishment projects with realistic sediment characteristics.

AVAILABILITY OF REPORT: The report is available through the Interlibrary Loan Service from the U.S. Army Engineer Waterways Experiment Station (WES) Library, telephone number (601) 634-2355. National Technical Information Service (NTIS) report numbers may be requested from WES Librarians. To purchase a copy of the report, call NTIS at (703) 487-4780.

About the Authors: The report was prepared and the associated research performed by Dr. Robert G. Dean, University Professor, and Mr. Jorge Abramian, Graduate Student, both of the Coastal and Oceanographic Engineering Department, University of Florida. Point of contact: Dr. Norman W. Scheffner, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, Principal Investigator for this work unit. For further information about the DRP, contact Mr. E. Clark McNair, Jr., Manager, DRP, at (601) 634-2070.

Dredging Research Program

Contract Report DRP-93-2
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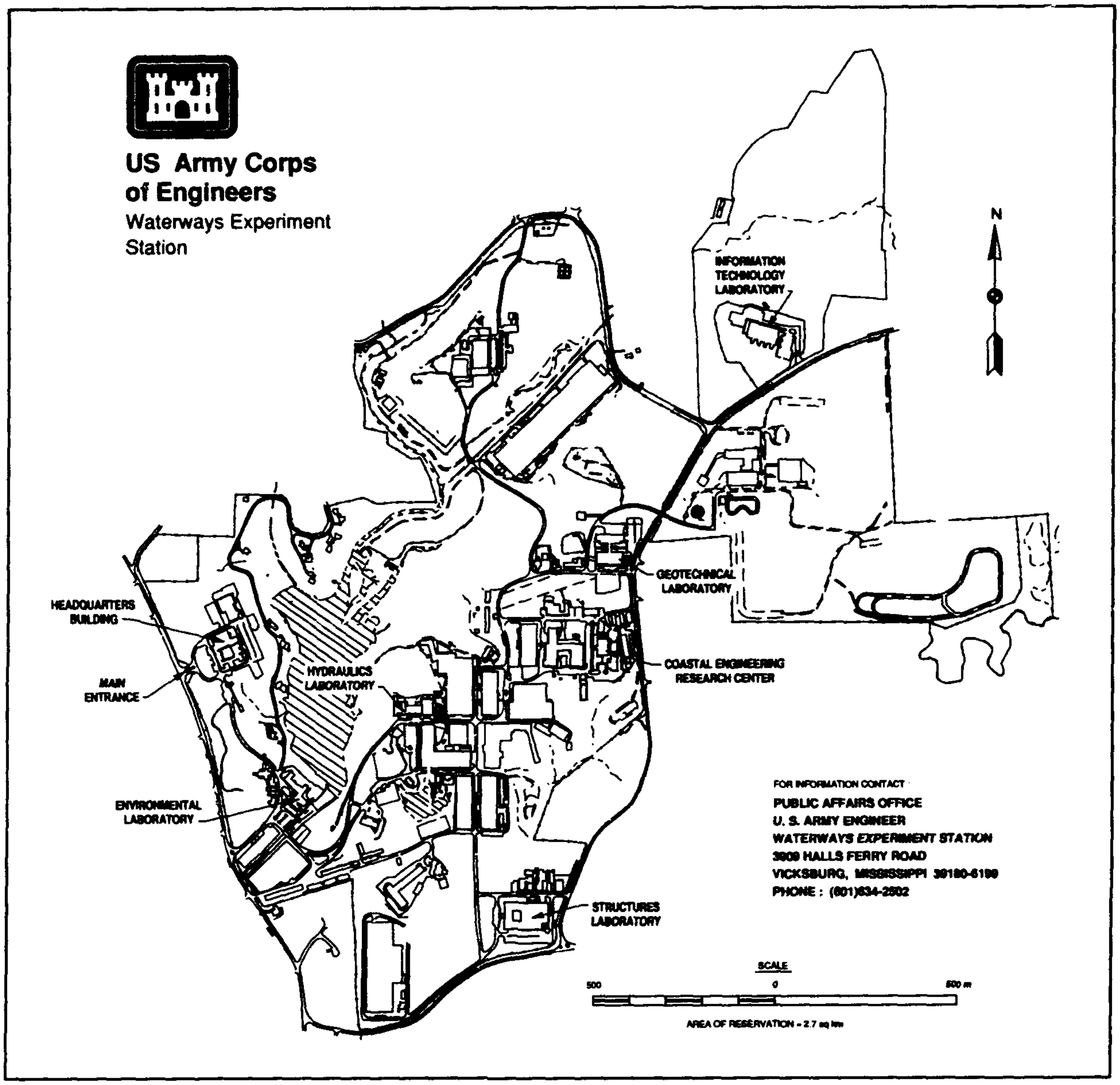

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Preface

The study reported herein results from research performed at the University of Florida (UF), Gainesville, Florida, under contract with the Dredging Research Program (DRP) of Headquarters, U.S. Army Corps of Engineers (HQUSACE). The contract was administered under the Calculation of Boundary Layer Properties (Noncohesive Sediments) Work Unit 32463, which is part of DRP Technical Area 1 (TA1), Analysis of Dredged Material Placed in Open Water. Messrs. Robert Campbell and John H. Lockhart, Jr., were DRP Chief and TA1 Technical Monitors from HQUSACE, respectively. Mr. E. Clark McNair, Jr., U.S. Army Engineer Waterways Experiment Station (WES) Coastal Engineering Research Center (CERC), was DRP Program Manager (PM), and Dr. Lyndell Z. Hales, CERC, was Assistant PM. Dr. Nicholas C. Kraus, Senior Scientist, CERC, was Technical Manager for DRP TA1 and Principal Investigator for Work Unit 32463.

This report was prepared and the associated research performed by Dr. Robert G. Dean, University Professor, and Mr. Jorge Abramian, Graduate Student, both of the Coastal and Oceanographic Engineering Department (COED), UF. Dr. Kraus provided technical review of the report and was under the administrative supervision of Dr. James R. Houston, Director, CERC and Mr. Charles C. Calhoun, Jr., Assistant Director, CERC. Ms. Cynthia J. Vey, Secretary, COED, typed the original manuscript, and Ms. Peggy T. Brown, Certified Professional Secretary, Secretary of Dr. Kraus, CERC, revised the report to WES format. Ms. Janean C. Shirley, Information Technology Laboratory, WES, was report text editor.

At the time of publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Leonard G. Hassell, EN.

Additional information on this report can be obtained from Mr. E. Clark McNair, Jr., DRP Program Manager, at (601) 634-2070 or Dr. Nicholas C. Kraus, Principal Investigator, at (601) 634-2018.

Summary

Rational design of beach nourishment projects requires the ability to calculate the geometry of the added sand volume. This capability is essential for quantitative evaluation of the relative merits of various borrow areas and for benefit/cost analysis of such projects. In many cases, the material may be a by-product of a dredging project carried out for purposes other than beach nourishment, and the dredged material to be placed may have a different grain size distribution than the original (native) beach.

This report presents a new methodology for predicting the equilibrium beach profile resulting from placement of an arbitrary volume of material with an arbitrary grain size distribution on a profile of arbitrary shape. The methodology developed, which depends on the theory of equilibrium profile shape, is proposed as an alternative to traditional compatibility and overfill ratio factors for borrow and native material. The methodology considers two-dimensional (cross-shore) conditions.

The theory is first developed and characteristics of equilibrium beach profiles relevant to beach nourishment projects are presented. The theory is then applied in four specific examples covering a range of beach, native sediment, and fill material conditions. A computer program developed for applying the method is given in an appendix.

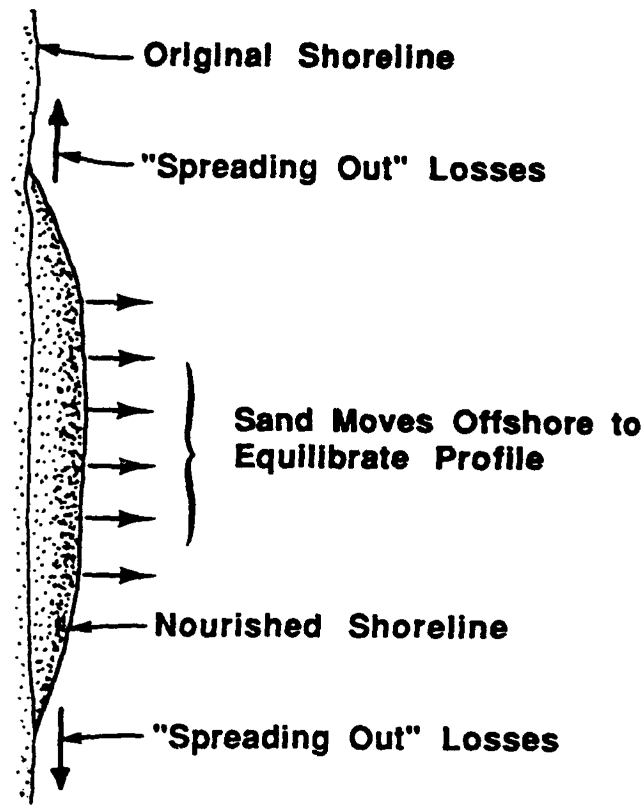
1 Introduction

Rational design of beach nourishment projects requires the ability to calculate the geometry of the added sand volume, both as a function of space and time. This capability is essential for quantitative evaluation of the relative merits of various borrow areas and in benefit/cost analysis of such projects, including the volumes and timing of renourishments. In many cases the material may be a by-product of a dredging project carried out for other purposes. Particular design aspects of significance include the equilibrated beach profile, especially the additional dry beach width and the longevity of the project. Traditionally, attempts to quantify benefits of beach nourishment have utilized "compatibility" and "overfill" factors, which relate the sand sizes of borrow and native material.

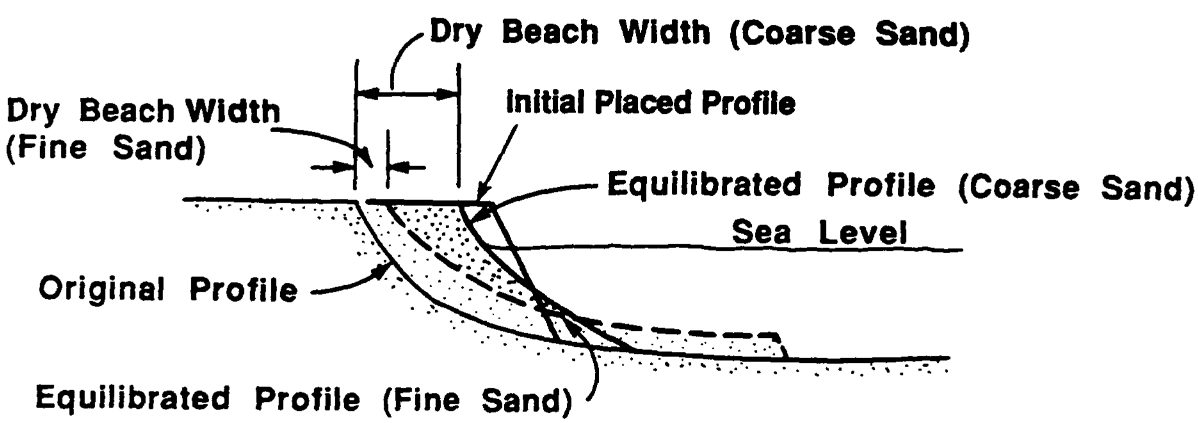
In a general sense, the problem of the *evolution of beach nourishment* projects can be considered as occurring over two more or less distinct time scales (see Figure 1). The *beach profile*, which is usually placed at a relatively steep slope, equilibrates over a fairly short time scale, perhaps with a "folding time" on the order of several years. The time scale associated with the *planform* evolution depends primarily on the length of the project and the wave climate; for longer projects (say greater than several kilometers), the time scale is on the order of decades. Regardless of whether these two time scales are distinct or not, it is useful to consider the equilibrium beach profile associated with the volume and texture (i.e., grain size distribution) of the nourishment material.

This report presents, for two-dimensional conditions, procedures for predicting the equilibrium beach profile resulting from placement of an arbitrary volume of material with an arbitrary grain size distribution on a profile of arbitrary shape.

This report is organized as follows: Chapter 2 reviews the background relative to efforts to quantify the suitability and/or effectiveness of materials for beach nourishment. Additionally, characteristics of equilibrium beach profiles relevant to beach nourishment are presented. These latter results pertain to perfectly sorted sediments. Chapter 3 describes and illustrates with examples the methodology developed in the present study for calculating equilibrium profiles for sediments of arbitrary sorting. A computer program



a) Plan View Showing "Spreading Out" Losses and Sand Moving Offshore to Equilibrate Profile



b) Elevation View Showing Original Profile, Initial Placed Profile, and Adjusted Profiles That Would Result from Nourishment Project with Coarse and Fine Sands

Figure 1. Sand transport losses and beach profiles associated with a beach nourishment project

developed for this purpose is described in terms of the general algorithms employed. Chapter 4 presents the results of a limited set of laboratory studies carried out in conjunction with this study. Chapter 5 describes relevant field data from Delray Beach, Florida, and Jupiter Island, Florida. Both areas have experienced multiple beach nourishment programs. Chapter 6 provides the summary, conclusions, and recommendations for further research. Appendix A is a list of the computer program provided to carry out calculations of equilibrium profiles for sediments of arbitrary sorting. Additionally, listings of the input and output files for two of the examples presented in this report are provided. Appendix B provides a detailed discussion of the computer program. Appendix C contains additional data for the Delray Beach, Florida, nourishment project. Appendix D is a notation of symbols used in this report.

2 Background

Various investigators have proposed procedures for relating the overall qualities of borrow and native sediments. As a rule of thumb, an attempt is made to locate borrow material with granulometric characteristics similar to those of the native material. Generally, the available procedures focus on comparing the grain size characteristics rather than on their response to a wave and tide regime. Thus these methods must be considered as ad hoc and not truly representative of the performance of a beach nourishment material. Moreover, these procedures do not address the additional dry beach width, a factor of primary concern to the designer and funding entities. A short review of the various available methods follows.

Krumbein and James (1965) proposed a method which considered the grain size distributions $f(\phi)$[1] of the borrow and native materials to be each represented by log-normal distributions as proposed earlier by Krumbein (1957)

$$f(\phi) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\phi-\mu)^2/2\sigma^2} \tag{1}$$

in which ϕ is the sediment diameter expressed in phi units, defined as

$$\phi = -\log_2(D(mm)) \tag{2}$$

where, as indicated, D is the sediment diameter in millimeters and μ and σ are the sample mean and standard deviation in phi units. This method defined compatibility of the borrow material on the basis of the proportion of borrow material distribution that was common with the native sand size distribution. This approach appears somewhat reasonable in discounting the finer fraction of the borrow material, but less reasonable in discounting similarly the proportion of coarse material that is in excess relative to the native sand.

[1] For convenience, symbols and abbreviations are listed in the notation (Appendix D).

James (1974) developed a complex method addressing the relative renourishment frequency for different sand characteristics; however, this procedure only considered *longshore* sediment transport and considered the nourishment project to be located in an area where the ambient longshore sediment transport had been interrupted completely.

Dean (1974) presented a method that attempted to address the deficiency (noted above) of the earlier Krumbein and James method. The borrow material was only discounted for the excessive proportion of fines present; excessive proportions of coarser material were included in the compatible fraction. However, it was considered that all the fine fraction smaller than a critical value was lost. This method resulted in a considerably higher compatibility than that of Krumbein and James (1965).

James (1975) developed a renourishment factor based on the relative characteristics of the borrow and native sand characteristics. Similar to earlier methods, this procedure was based on size distributions rather than associated equilibrium profiles. Compared to Dean's method (1974), the primary difference is the retention of a portion of the fine fraction in the compatibility considerations. At present, the methods of James (1975) are those recommended in the *Shore Protection Manual* (1984).

Companion to the problem of defining borrow material compatibility is that of sampling across the pre-nourishment profile to establish the "native" sand characteristics. This problem has been addressed by several investigators.

Although the primary focus of the present report is the equilibrium beach profile for sand of arbitrary distribution, it is useful to consider, for background purposes, profiles which result for the idealized case of uniform borrow and native sediment sizes.

Dean (1991) has considered equilibrium beach profiles represented by $h(y) = Ay^{2/3}$ first proposed by Bruun (1954) and found later by Dean (1977) in an analysis of more than 500 profiles extending from the eastern end of Long Island around Florida to the Gulf of Mexico border. In this equation h is water depth, y is distance seaward from the shoreline, and A is an empirical coefficient called the scale parameter or simply "A value." Moore (1982) investigated the relationship between the sediment scale parameter A and the sand diameter D, and established the results shown by the curved line in Figure 2. Later Dean (1987) simply transformed this A versus D relationship to A versus w where w is the fall velocity. The result was found to be well-approximated by the straight line in Figure 2.

It has been shown that three types of profiles could occur, depending on the relative sizes of the borrow and native sands. These are termed "intersecting," "non-intersecting," and "submerged" profiles and are illustrated in Figure 3. The reader is referred to the paper of Dean (1991) for the criteria separating the three profile types and the volumes required to achieve, for

example, a desired additional dry beach width of the nourished profile (for intersecting and non-intersecting profiles).

The significance of these three profile types can be seen by referring to Figures 4, 5, and 6. Figure 4 shows the effect on additional dry beach width of placing the same volume (340 m^3/m) of sand of four different grain sizes. In the upper panel, the sediment is coarser than the native and the profiles intersect with an additional dry beach width of 92.4 m. Panel b shows the effect of using sand of the same size as the native resulting in a dry beach width of 45.3 m. Panels c and d present the results for decreasing sediment size; in Panel d, the dry beach width is zero.

Figure 5 shows the effect of nourishing with various quantities of a sediment that is smaller than the native. With the same sediment size, the volumes of sediment increase from Panels a to d. With increasing volume, the landward and seaward extent of the nourished regions increase and in Panel d, sufficient volume has been added to achieve a transition between submerged and non-intersecting profiles.

These types of results can also be presented as shown in Figure 6 for an example that is in a form more representative of beach nourishment concerns. This figure shows the relationship of additional dry beach width Δy versus volume added V for three values of nourishment sediment scale parameter A_F. Other variables common to the three cases are: berm height, B = 2.0 m, depth of effective motion, h_* = 8.0 m, and native sediment scale parameter, $A_N = 0.1\ m^{1/3}$. Of interest is that for $A_F = 0.12 > A_N$, the profiles are initially intersecting and the additional dry beach width increases relatively rapidly. However, with increasing volume, the profile becomes non-intersecting and the slope $d(\Delta y)/dV$ is approximately the same as that for $A_F = A_N$, which is almost a constant. For $A_F = 0.08\ m^{1/3} < A_N$, for small volumes of sediment, there is no additional beach width, i.e., the profile is submerged. However, with increasing volumes, there is a critical volume at which the landward end of the submerged profile just reaches the shoreline; for still greater volumes, the profile becomes a non-intersecting profile and remains so for increasing volumes. For this case, the slope $d(\Delta y)/dV$ is essentially constant and parallel to the case of $A_F = A_N$. It is stressed that all of these results apply for *equilibrium* profiles and that the equilibration process may take several years to complete. In most nourishment projects, the sand is placed steeper than equilibrium and will provide greater additional dry beach width than equilibrium during the equilibration process.

In general it can be shown (Dean 1991) that the non-dimensional additional beach width $\Delta y/W_*$ is related to the non-dimensional volume added V/BW_*, non-dimensional berm height B/h_*, and ratio of fill to nourishment sediment scale factors A_F/A_N, i.e.,

$$\frac{\Delta y}{W_*} = f\left(\frac{V}{BW_*}, \frac{B}{h_*}, \frac{A_F}{A_N}\right) \tag{3}$$

where h_* and W_* are the depth of limiting motion and width of the active profile for the initial native profile. Figures 7 and 8 present this relationship for $B/h_* = 1/2$ and $1/4$, respectively. Several features of these figures are of interest. First, for $A_F/A_N > 1.2$ (approximately), there is little additional dry beach width gained for coarser (greater A_F/A_N) material used. Secondly, for $A_F/A_N < 1$, and a fixed volume, there is a rapidly decreasing dry beach width with decreasing A_F/A_N. Finally, the transition from intersecting to non-intersecting profiles is indicated by the bold line in Figures 7 and 8 and the transition from non-intersecting profiles to submerged profiles occurs at the vertical asymptotic lines at the left end of each of the curves.

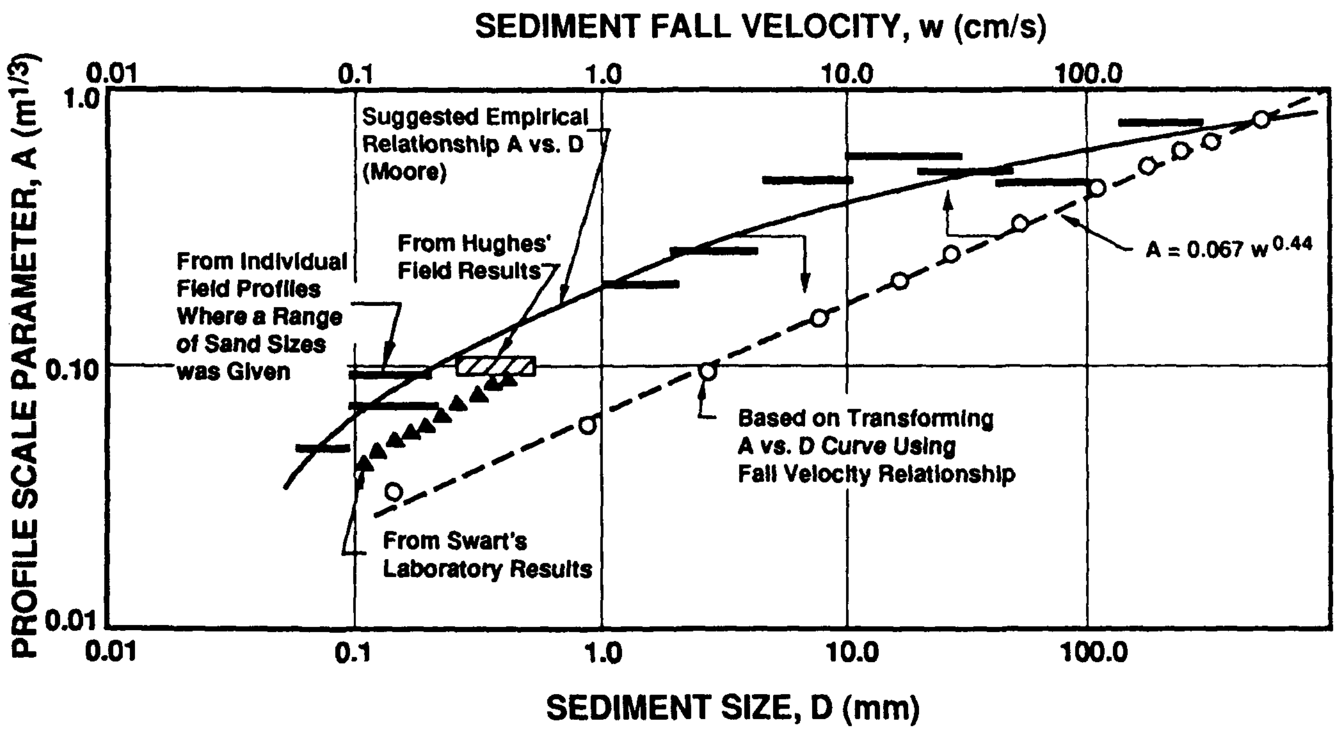


Figure 2. Variation of sediment scale parameter *A* with sediment size and fall velocity

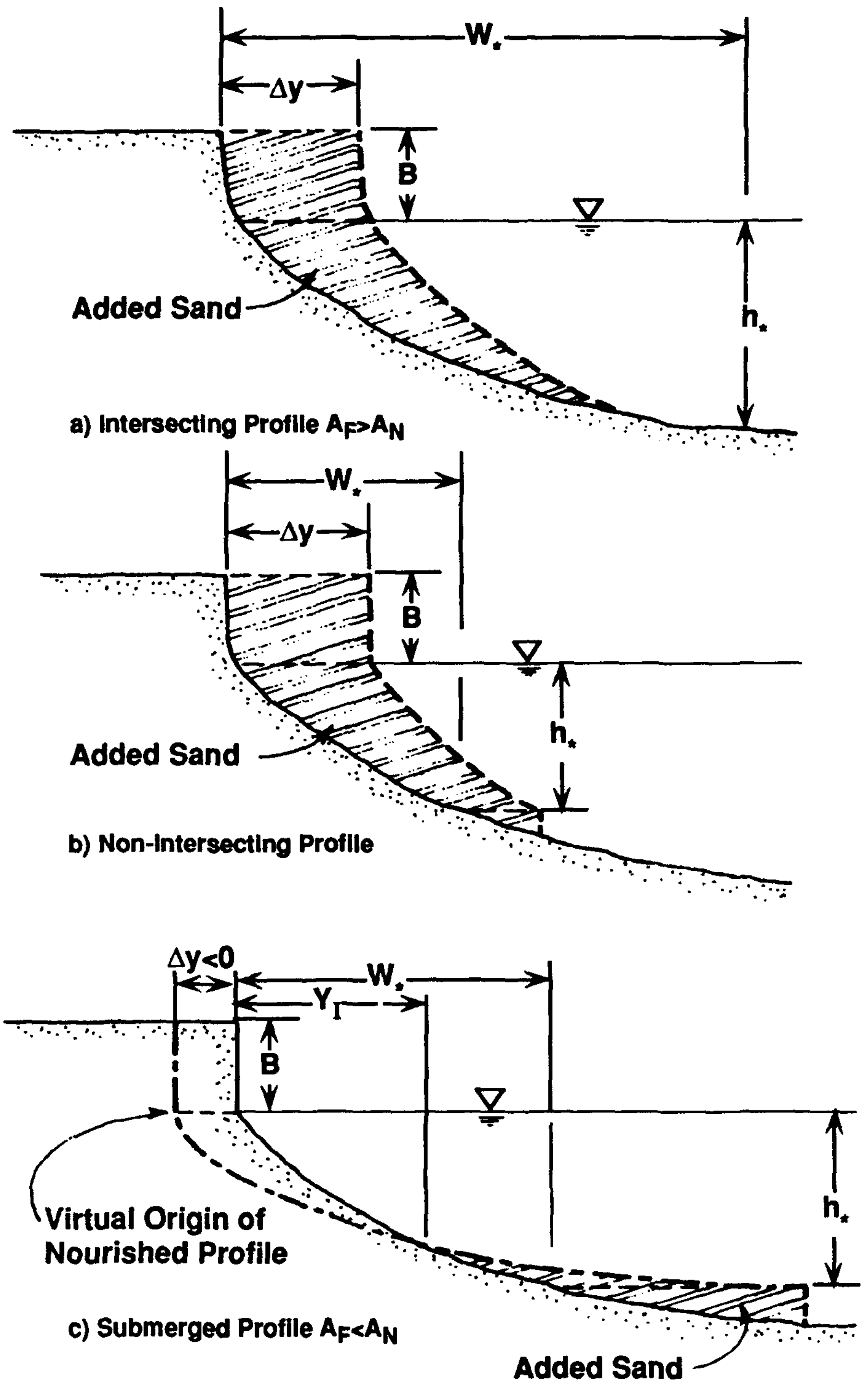


Figure 3. Three generic types of nourished profiles: (a) intersecting profile, (b) non-intersecting profile, and (c) submerged profile

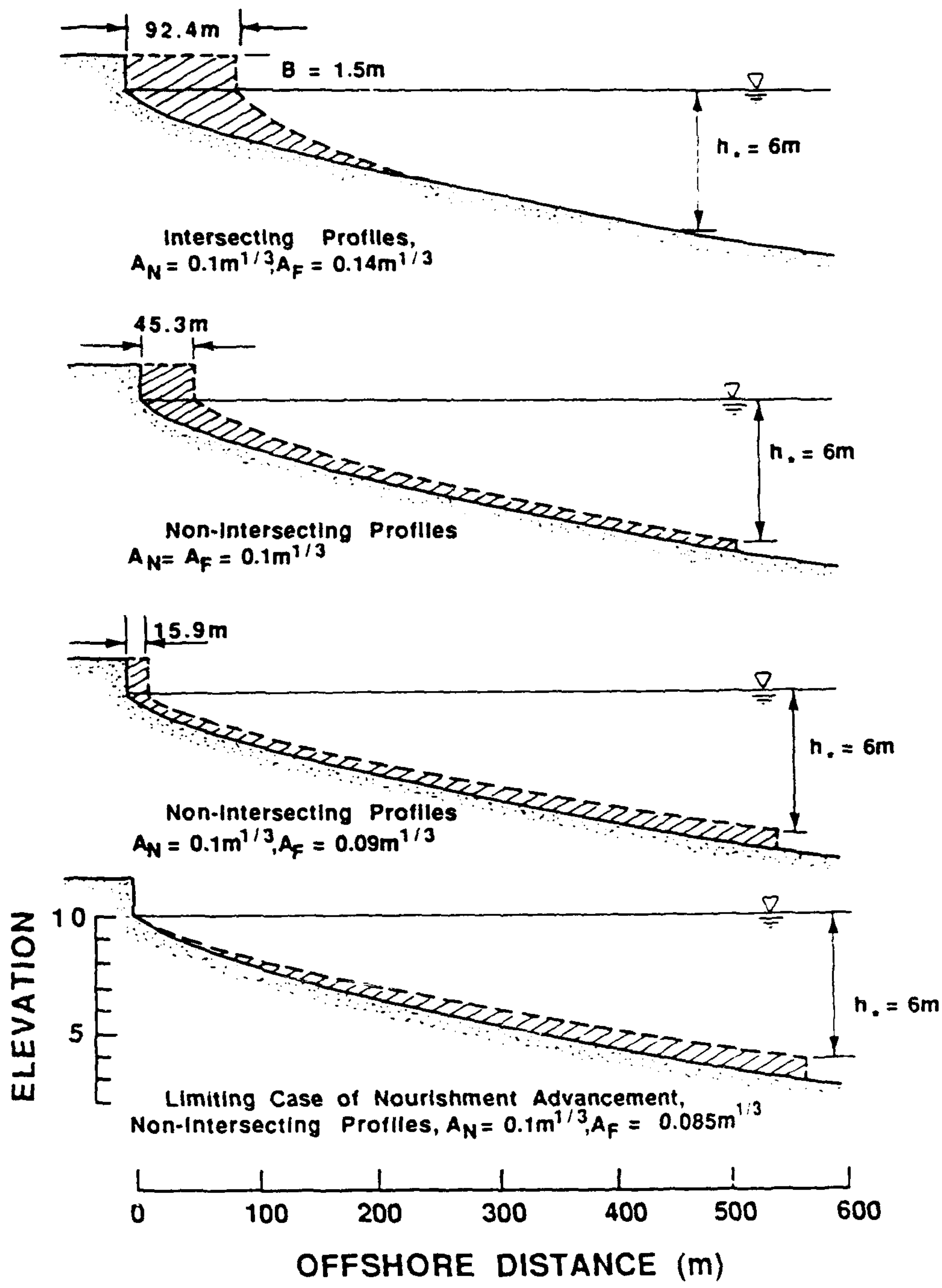


Figure 4. Four examples of decreasing A_F with same added volume per unit beach length

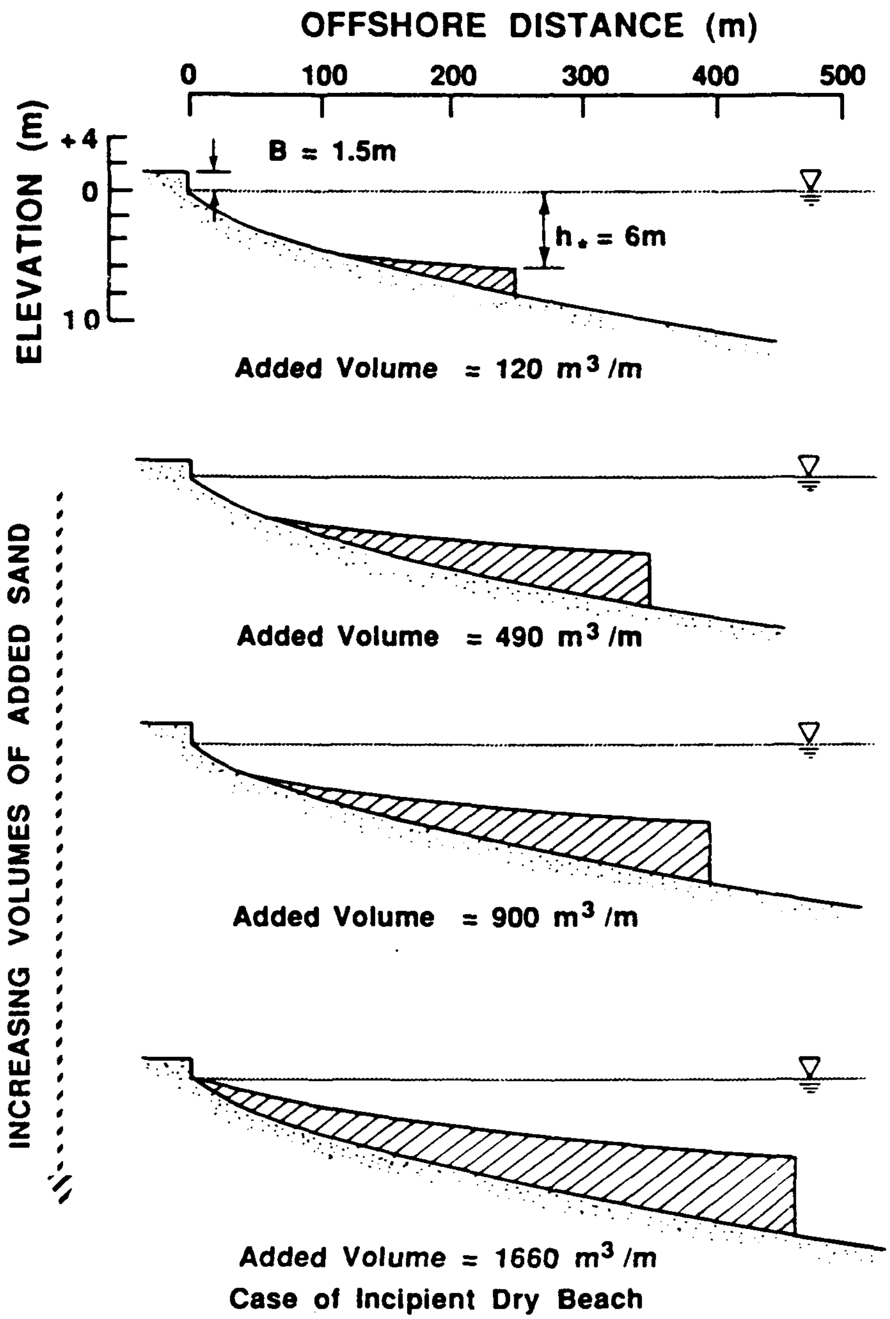


Figure 5. Effect of increasing volume of sand added on resulting beach profile. $A_F = 0.1\ m^{1/3}$, $A_N = 0.2\ m^{1/3}$, $h_* = 6.0$ m, $B = 1.5$ m

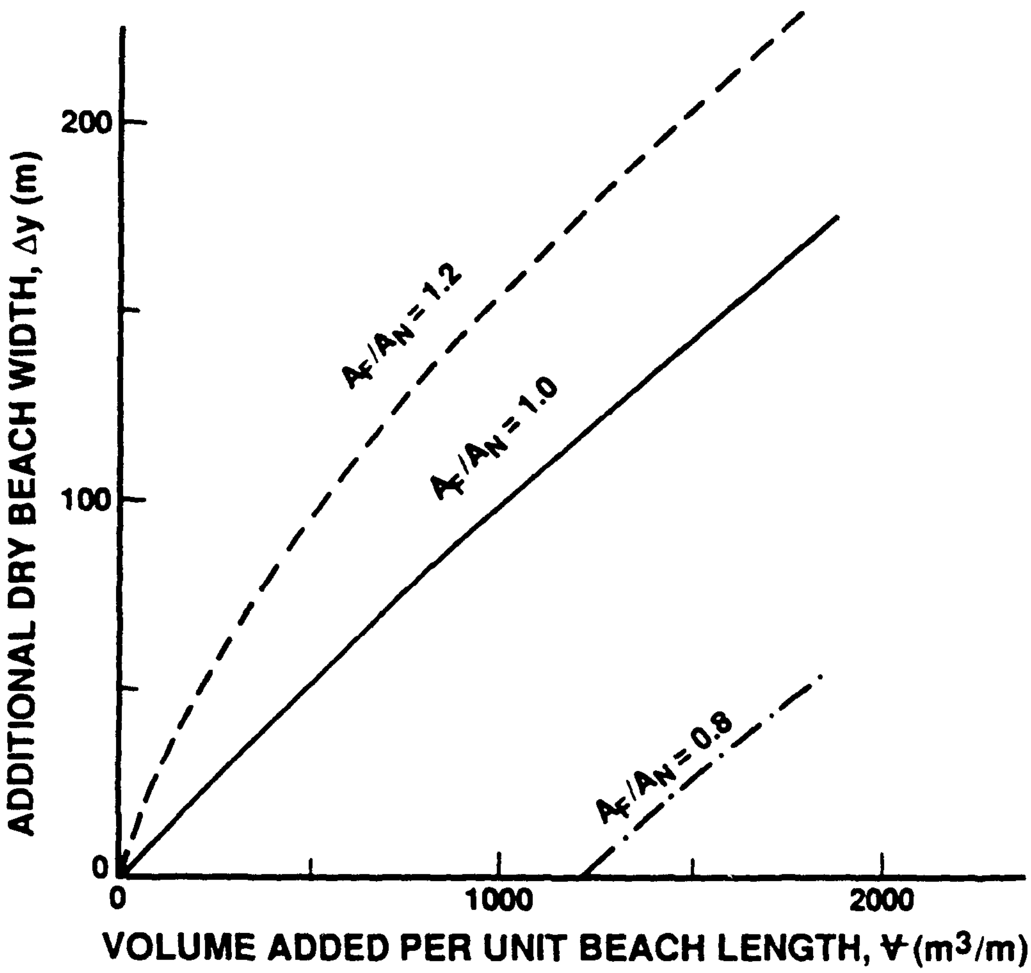


Figure 6. Effect of volume added $\forall$ and fill sediment scale parameter A_F on added dry beach width Δy

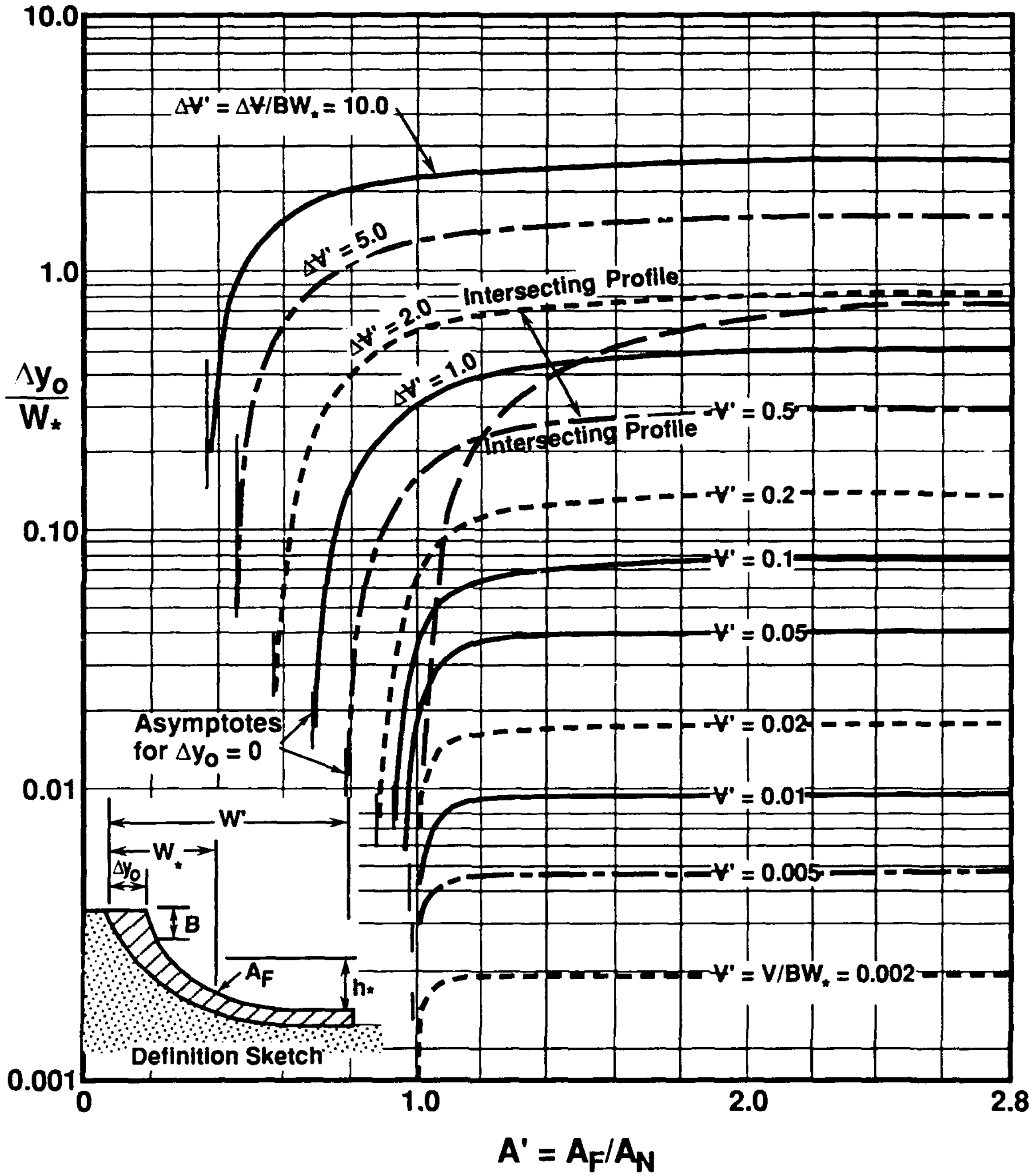


Figure 7. Variation of non-dimensional shoreline advancement $\Delta y/W_*$ with A' and V. Results shown for $h_*/B = 2.0$

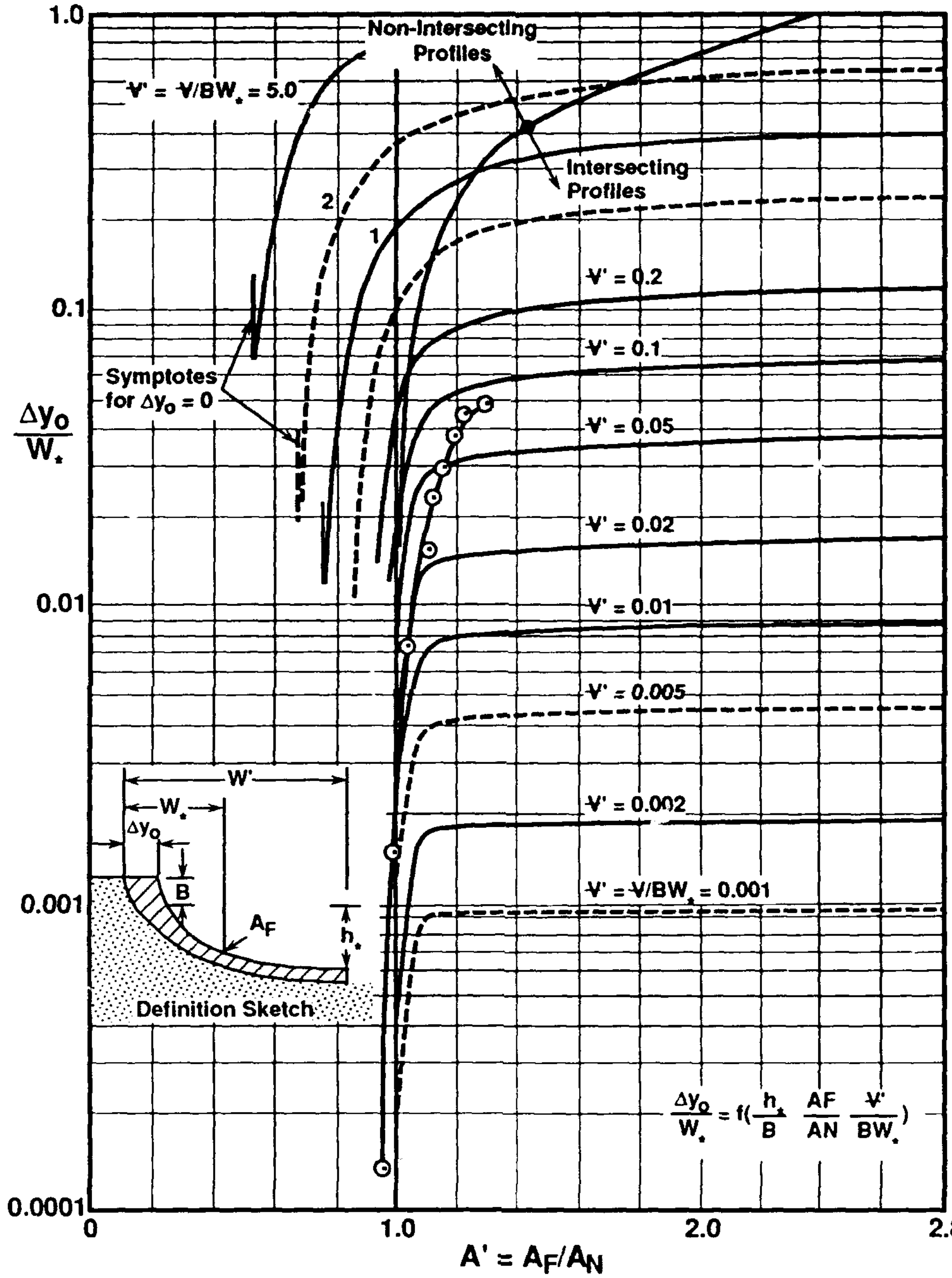


Figure 8. Variation of non-dimensional shoreline advancement $\Delta y/W_*$ with A' and $\forall$. Results shown for $h_*/B = 4.0$

3 Methodology Based on Equilibrium Beach Profiles

The method developed herein is based on an equilibrium profile for a sediment volume with a distribution of sediment sizes. The following assumptions/considerations are made:

a. A volume of sand $\forall$ per unit beach length is placed at a slope steeper than equilibrium in conjunction with a beach nourishment project.

b. The sand is well-mixed at the time of placement.

c. This sand will be reworked such that the volume removed from the placement cross section is sufficient to extend the nourished equilibrium profile out to a specified depth h_* of limiting motion (see Figure 9) or to its intersection with the initial profile.

d. Within the zone of sediment removal from the placement cross section, sorting occurs down to a specified thickness Δh_{mix}.

e. The available sediment is sorted across the profile with the coarser fraction remaining in the berm and shallower water region and the finer sediment distributed offshore.

f. The volumes of sediment removed and deposited are equal; i.e., volume is conserved.

With the above basis, the procedure can be considered as one of locally establishing segments of an equilibrium profile consistent with the local A value and of balancing sediment volumes. Because the equilibrium beach profile form $h = Ay^{2/3}$ yields an unrealistic infinite slope at $y = 0$, the modified form was used, which recognizes the effect of gravity for the larger slopes

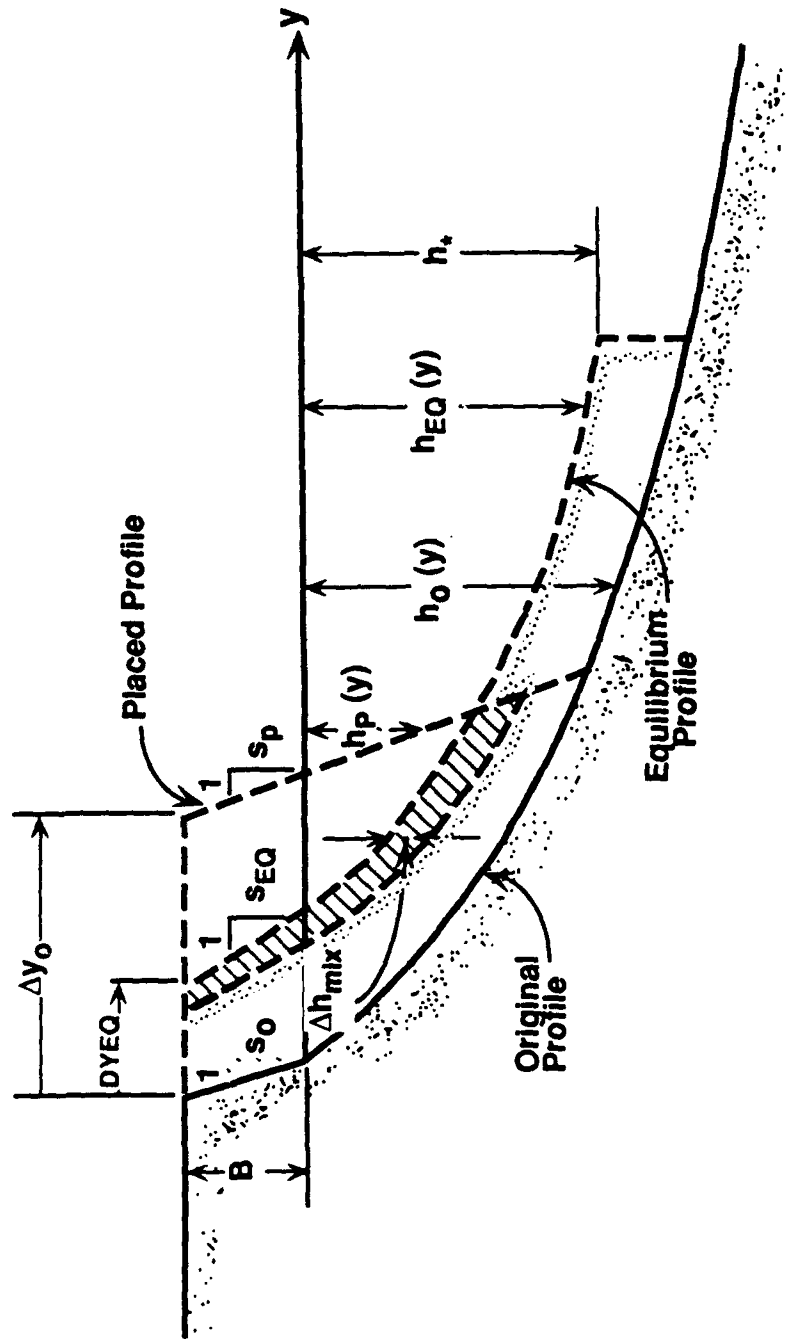


Figure 9. Definition sketch

$$y = \frac{h}{s} + \frac{h^{3/2}}{A^{3/2}} \tag{4}$$

as initially proposed by Dean (1983) and later shown by Larson (1988) and Larson and Kraus (1989) to be derivable from the breaking wave model of Dally, Dean, and Dalrymple (1985) under the consideration of uniform wave energy dissipation per unit volume. In Equation 4, s is the beach face slope. It can be shown easily that in shallow water,

$$h \approx sy \tag{5}$$

i.e., the beach is planar, consistent with measurements in nature. In deeper water, Equation 4 approximates $h = Ay^{2/3}$.

Because the A value is now *local*, the depth at a location $y + dy$ is referenced to the depth at y based on Equation 4,

$$h(y+dy) = h(y) + \frac{dy}{(\partial y/\partial h)} \tag{6}$$

where

$$\frac{\partial y}{\partial h} = \begin{Bmatrix} \frac{1}{s} & , h < 0 \\ \frac{1}{s} + \frac{3}{2}\left(\frac{h^{1/2}}{A^{3/2}}\right)_{h(y)} & , h > 0 \end{Bmatrix} \tag{7}$$

and the dy values are maintained reasonably small, on the order of 1-2 m.

A step-by-step discussion of the procedure is as follows and is illustrated in the program flowchart, Figure 10.

a. With the specified initial profile $h_0(y)$, added volume $\forall$, berm height B, and placement slope s_p, the placed profile $h_p(y)$ is determined by iteration such that the volume out to the location where $h_p(y) = h_0(y)$ is the volume placed. This procedure also determines the berm advancement Δy_o.

b. Trial values of the volume sorted $\forall_{GEN}$ and equilibrated berm advancement DYEQ are assumed (refer to Figure 9 for definition of variables). For each pair of these quantities, the equilibrium profile is advanced from y to $y + dy$, where dy is constant, say 1-2 m. This advancement is in accord with Equations 6 and 7. The local A is that associated with the diameter for the coarser fraction of the sediments that has not been deposited up to y in the equilibration process (see Figure 11). This

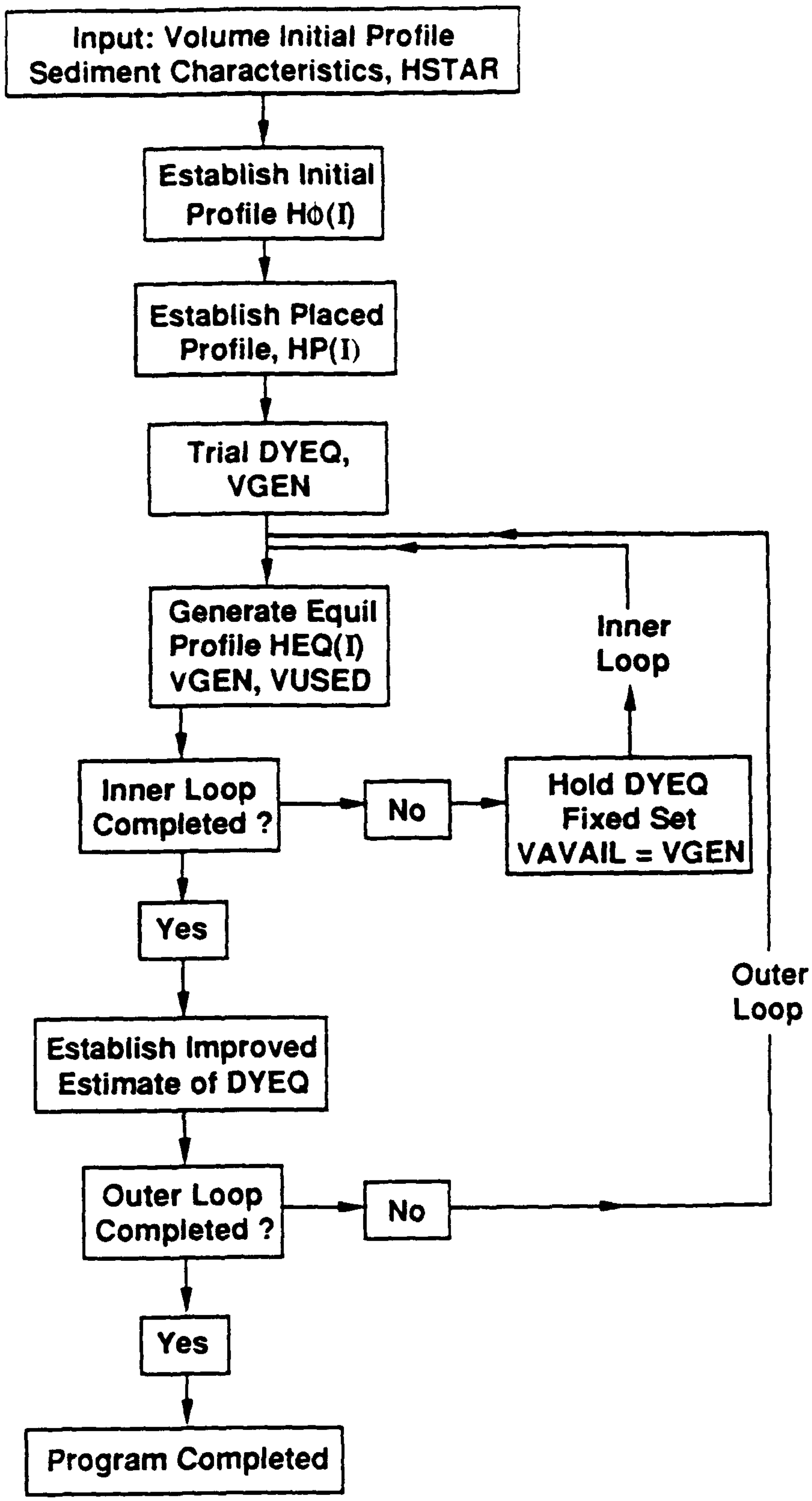
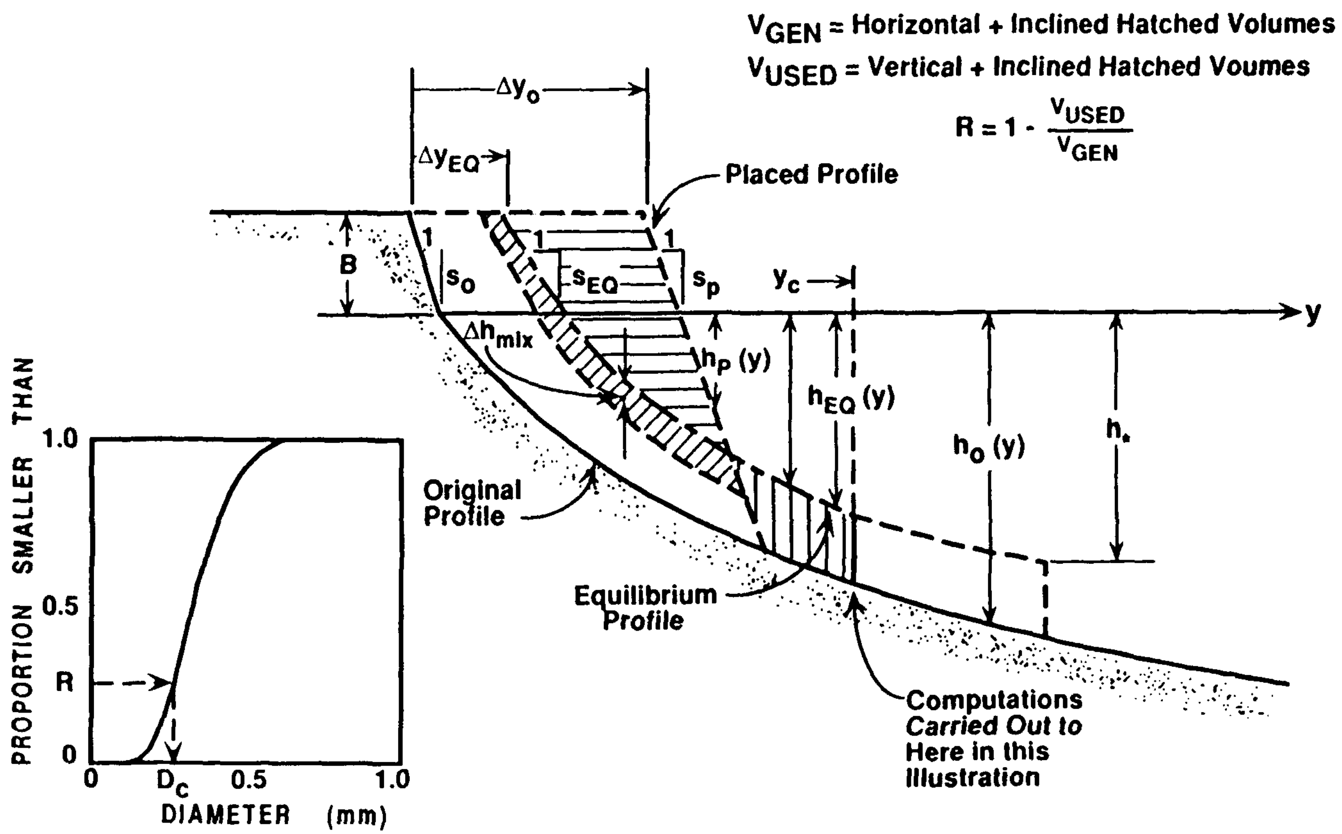


Figure 10. Flow diagram for problem solution



(b) Cumulative Grain Size Distribution

(a) Original, Placed and Equilibrium Profiles, The Latter Carried Out to y_c

Figure 11. Method of determining grain size D_c for computation of current increment of equilibrium profile

step-by-step advancement is continued until the depth equals the specified terminal depth h_* or until the equilibrium profile intersects with the initial profile. At that stage, the volume actually generated through erosion of the placed profile is substituted for the volume available and the equilibrated berm advancement DYEQ is held fixed in this inner loop and the process repeated. This inner loop (with DYEQ) fixed is repeated until V_{GEN} values in two successive iterations agree within an acceptable limit.

c. The value of DYEQ is changed to attempt to ensure that the associated value of $h = h_*$ or profile intersection will be achieved coincident with the deposition of VGEN for that value of DYEQ. The DYEQ at the $k+1$ iteration is based on the following simple algorithm

$$DYEQ^{k+1} = DYEQ^{k} + F^{k+1}(\Delta DYEQ) \tag{8}$$

in which ΔDYEQ is specified as some reasonable value, say 2 or 5 m, and $F^1 = +1$, $F^2 = \pm 1$ for $k=2$ and the positive and negative signs apply depending on whether $V_{GEN} > V_{USED}$ ($F^2 = +1$) or $V_{GEN} < V_{USED}$ ($F^2 = -1$). In subsequent iterations ($k>2$), $F^{k+1} = F^k$ if the sign of $V_{GEN} - V_{USED}$ did not change in the preceding iteration and $F^{k+1} = -0.5\ F^k$ if a sign change did occur.

It is noted that the solution procedure structure is identical for both idealized and arbitrary grain size distributions. In this context, idealized refers to grain size distributions given by Equation 1. Additionally, the method can be applied for arbitrary initial profiles.

Examples

Methods developed in the earlier sections of this paper will be illustrated with examples.

Example 1

Idealized initial profile and log-normal size distribution, non-intersecting profile:

In this example, the initial profile was specified as characterized by the following:

Uniform sand size: $A = 0.1\ m^{1/3}$ ($D = 0.20$ mm)
Berm height: $B = 1.5$ m
Beach face slope: $s_o = 1{:}10$

The characteristics of the nourishment material are as follows:

Volume added: 140.0 m^3/m
Log-normal sand size: $\mu = 1.6\phi$ ($D = 0.33$ mm), $\sigma = 0.40\phi$
Berm height: $B = 1.5$ m
Placed slope : $s_p = 1{:}10$
Equilibrium beach face slope: $s_{EQ} = 1{:}20$
Mixed depth: $\Delta h_{mix} = 0.2$ m
Depth of active motion: $h_* = 6$ m

Figure 12 presents the size distribution of the sediment. The volume and slope above yielded a placed shoreline advancement Δy_o of 58.2 m.

Figure 13 shows the initial, placed, and equilibrium beach profiles. It is seen that for this case the equilibrated shoreline advancement is 20.3 m. The volume eroded for this case is 66.8 m^3 and, of course, the volume eroded is equal to the volume deposited. For this example, the equilibrium profile extends to an offshore distance of 550 m, where it reaches the specified depth of 6 m.

Example 2

Idealized initial profile and log-normal size distribution, intersecting profiles:

The characteristics of this example are the same as for Example 1 except the sorting coefficient σ of the placed sand is 0.1ϕ rather than 0.4ϕ. The cumulative sediment size distribution is shown in Figure 12. For this case the equilibrium and original profiles intersect at a depth of 4.69 m, which is located at a distance of 425 m offshore. The initial, placed, and equilibrium profiles are presented in Figure 14.

Example 3

Sand smaller than native, near-zero shoreline advancement:

The initial profile was specified as follows:

Uniform sand size: $A_N = 0.1\ m^{1/3}$ ($D = 0.2$ mm)
Berm height: $B = 1.5$ m
Beach face slope: $s_o = 1{:}10$

The characteristics of the nourished profile are:

Volume added: 240 m^3/m
Log-normal sand size: $\mu = 2.64\phi$ ($D = 0.16$ mm), $\sigma = 0.10\phi$ (see Figure 12)
Berm height: $B = 1.5$ m

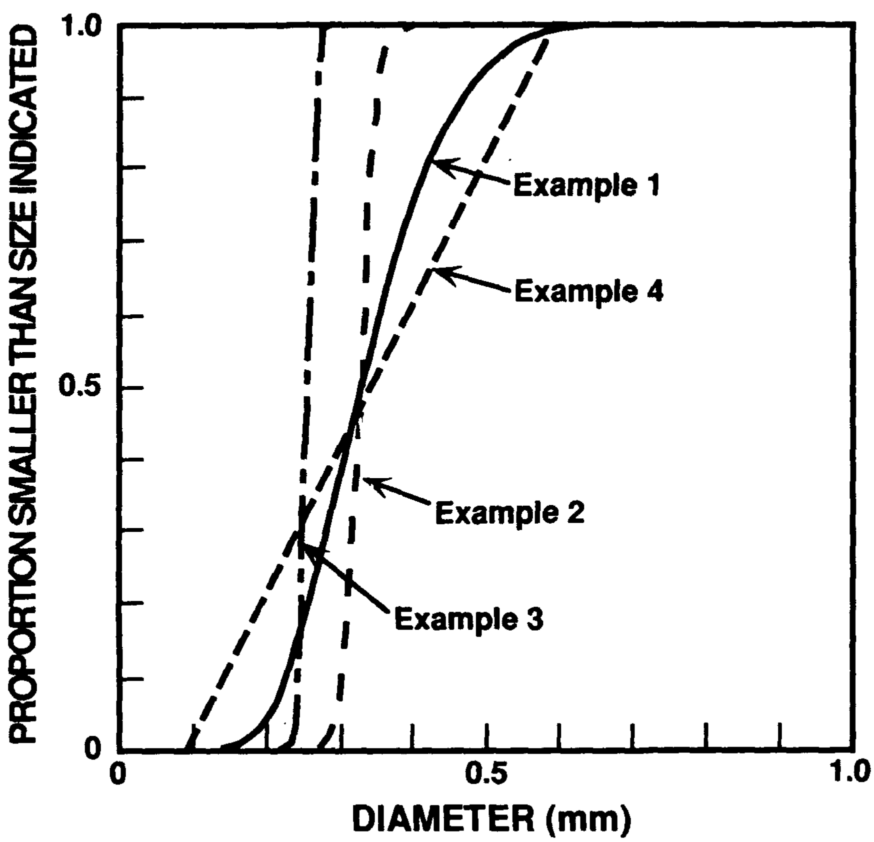


Figure 12. Cumulative grain size distribution, Examples 1 through 4

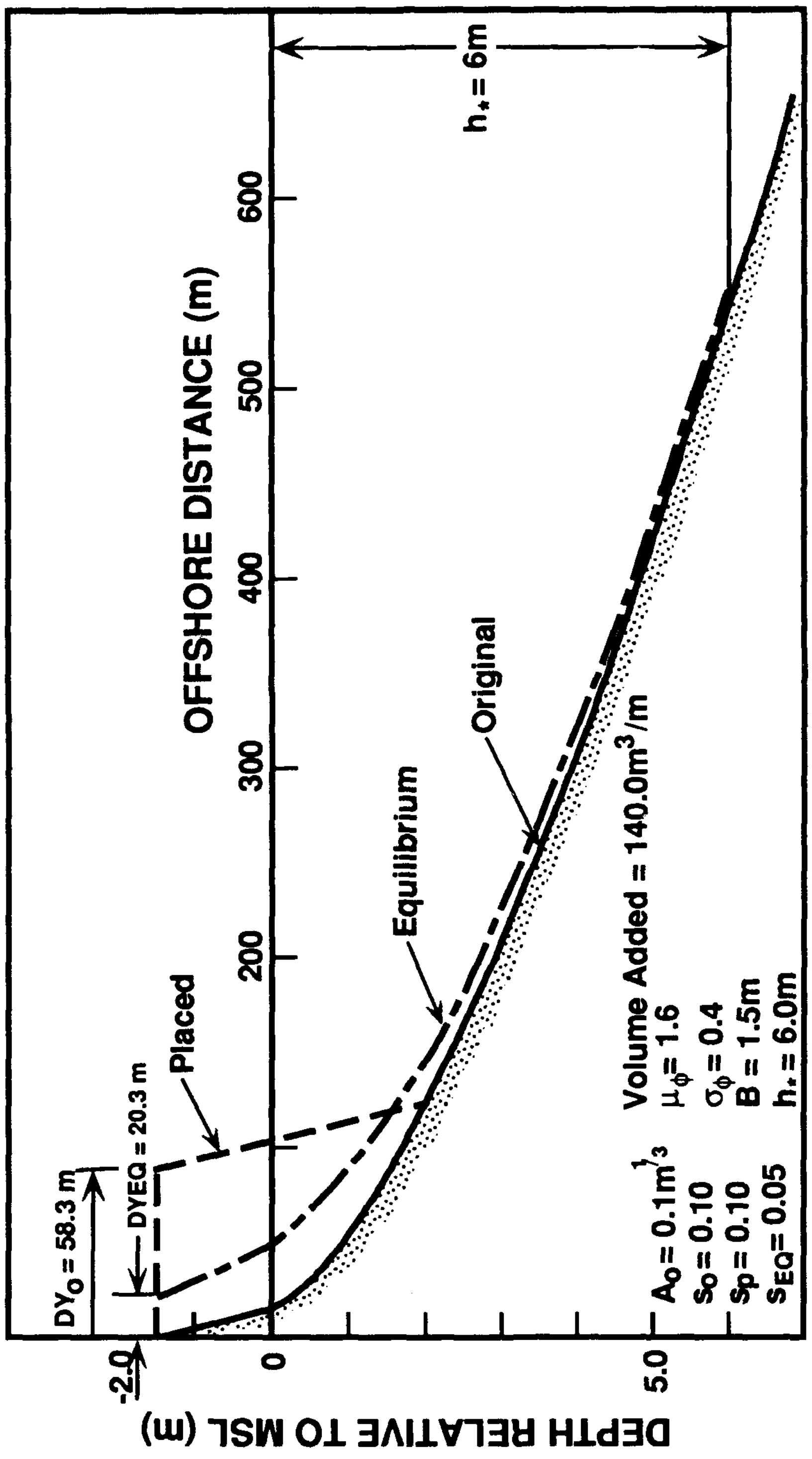


Figure 13. Example 1, original, placed, and equilibrium profiles. Case of non-intersecting profiles. Idealized grain size distribution

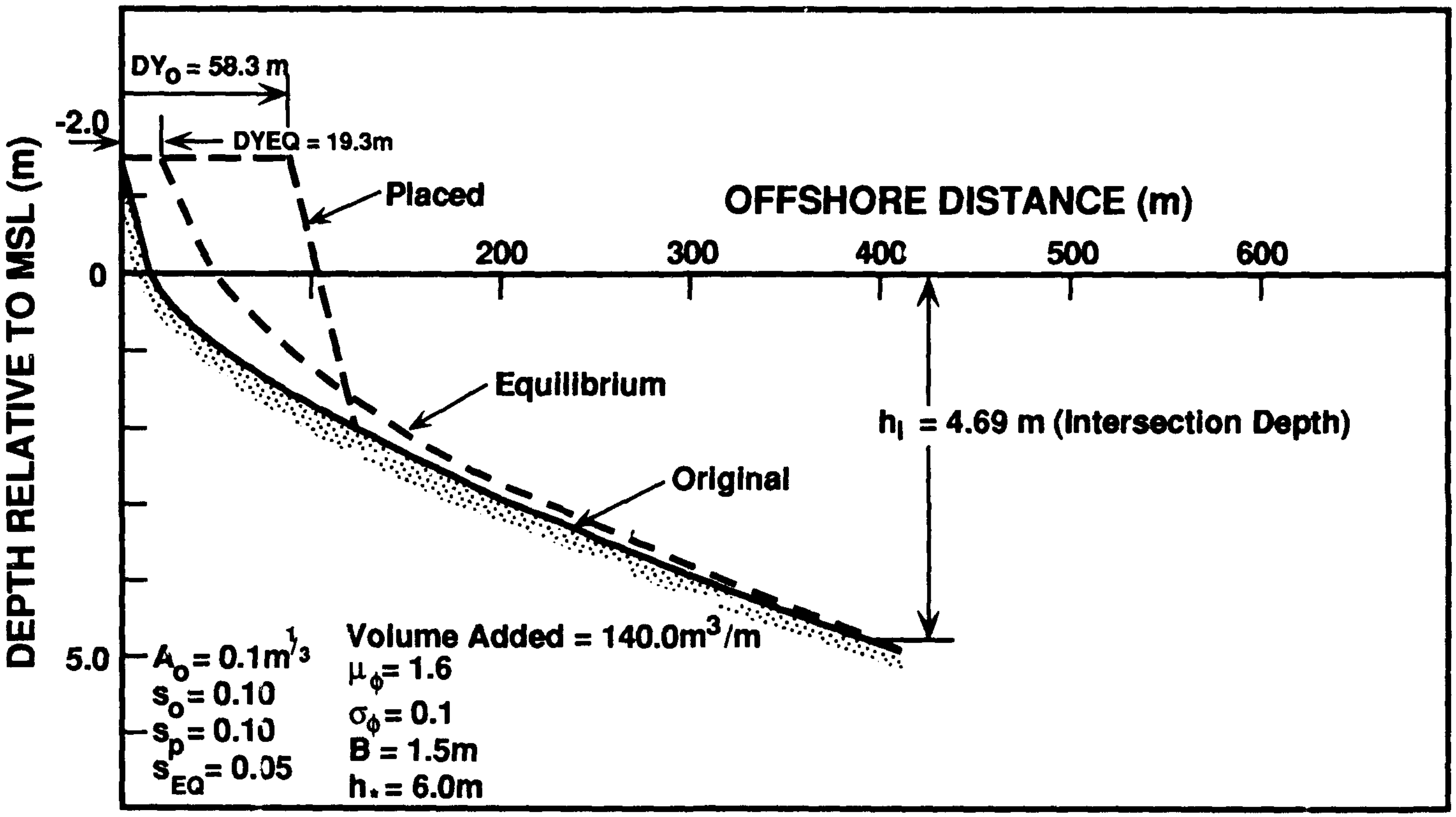


Figure 14. Example 2, original, placed, and equilibrium profiles. Case of intersecting profiles. Idealized grain size distribution

Placed slope: $s_p = 1{:}10$
Equilibrium beach face slope: $s_{EQ} = 1{:}10$
Mixed depth: $\Delta h_{mix} = 0.2$ m
Depth of active motion: $h_* = 6$ m

This example, presented in Figure 15, illustrates conditions near a transition to a submerged profile.

Example 4

User-specified initial beach profile and sediment characteristics:

In this example, the initial beach profile was specified by nine points. Between these points, the profile is considered as a series of straight line segments. The sediment size distribution is specified as linear as shown in Figure 12. A sediment volume of 600 m^3/m has been added. Other variables are similar to those specified in Example 3 and are shown along with the initial, placed, and equilibrium profiles in Figure 16. The equilibrium profile is of the non-intersecting type; however, intersection nearly occurs. Other characteristics are similar to those in the three previous examples.

Example 5

Variation of additional dry beach width versus volume added:

Figure 17 presents the variation of additional dry beach width versus nourishment sand volume added for three different grain sizes. The common characteristics are:

$A_o = 0.10\ m^{1/3}$
$s_o = s_p = s_{EQ} = 1{:}10$
$\Delta h_{mix} = 0.2$ m
$B = 1.5$ m
$h_* = 6$ m

These results are in the form of Figure 6, which was based on perfectly sorted sediment. For the upper curve in Figure 17, which applies for sediment coarser than the native, there is a transition from intersecting to non-intersecting profiles at an added volume of 600 m^3/m. For the other two sediments, *all* equilibrium profiles are non-intersecting.

Summary and Conclusions for Methodology Presented

Based on earlier work, three distinct types of equilibrium profiles can exist: non-intersecting, intersecting, and submerged.

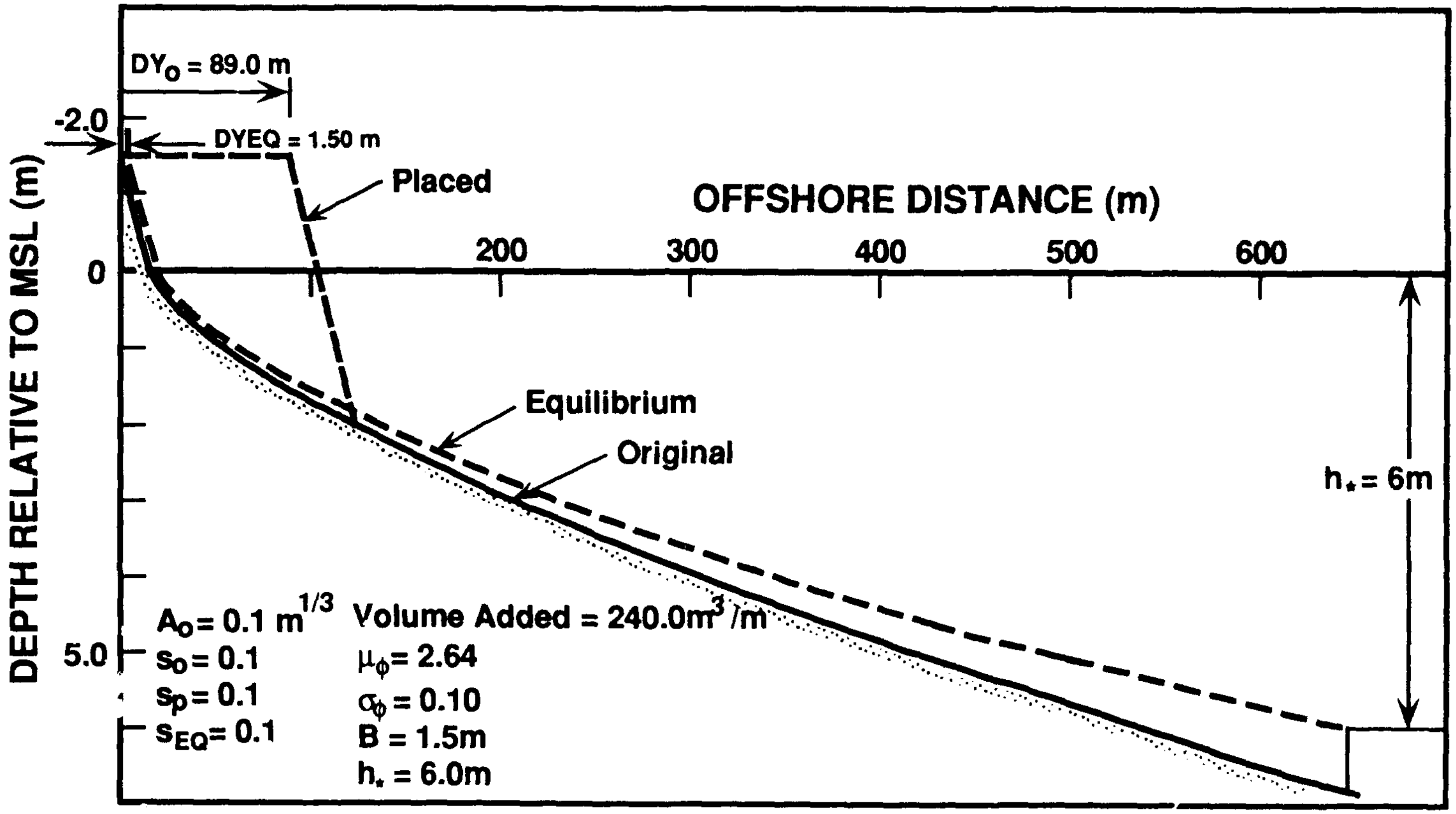


Figure 15. Example 3, original, placed, and equilibrium profiles. Case of non-intersecting profiles. Idealized grain size distribution smaller than native. Incipient submerged profile

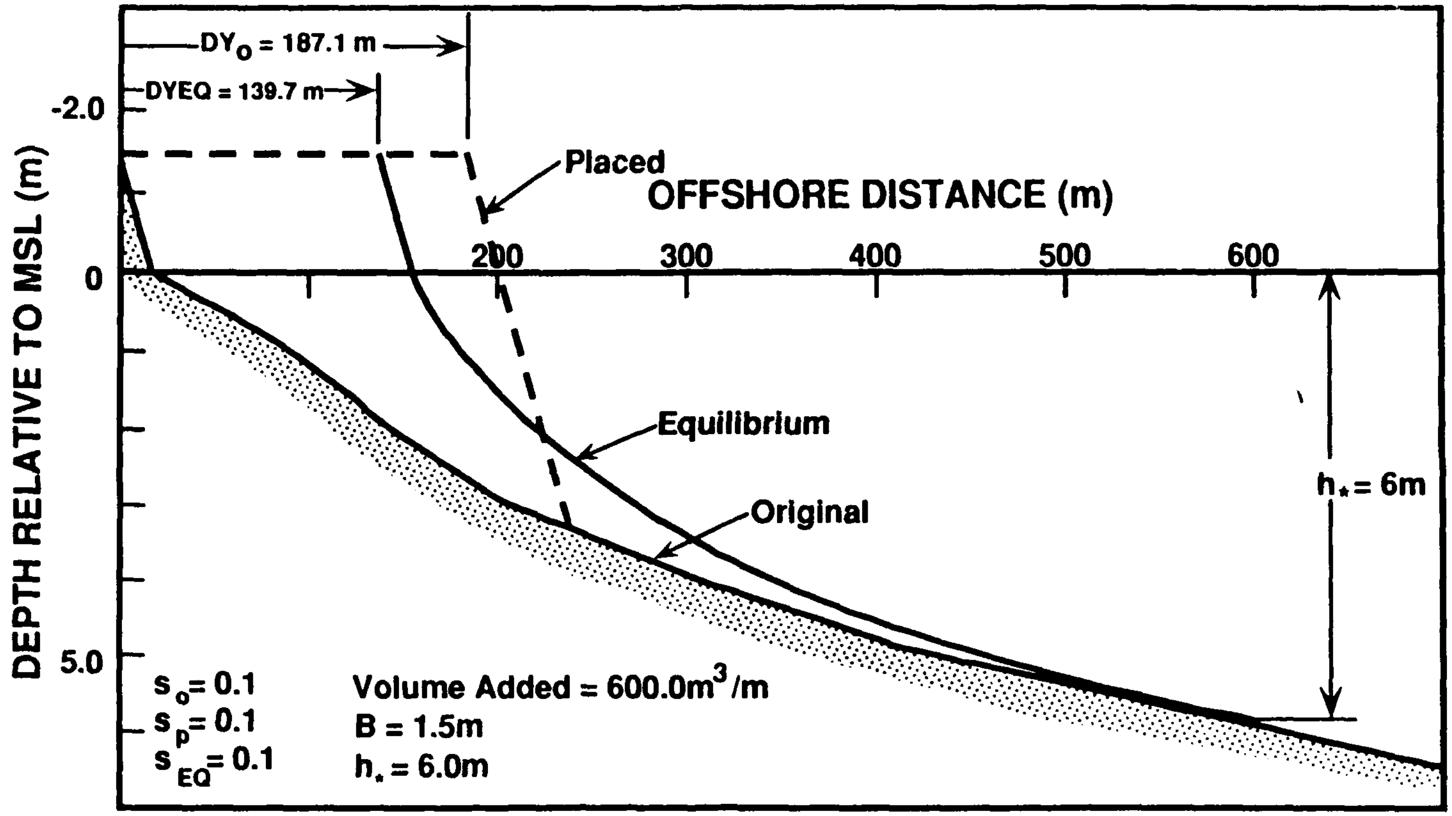


Figure 16. Example 4, original, placed, and equilibrium profiles. Case of non-intersecting profiles for user-specified original profile and nourishment material grain size distribution

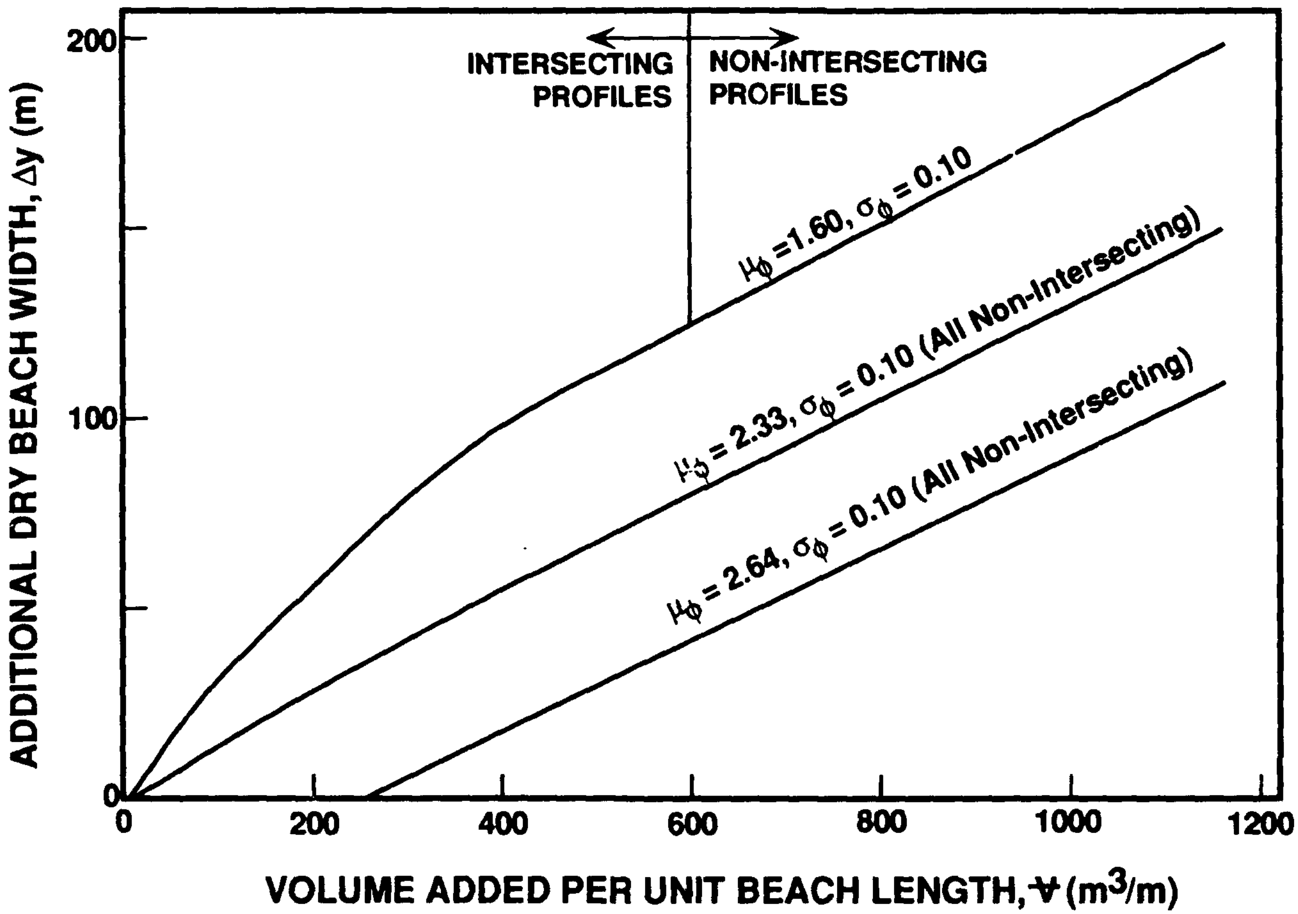


Figure 17. Additional dry beach width Δy versus nourishment volume per unit beach length $\forall$. Three nourishment materials

Methods have been developed and illustrated with examples to calculate the non-intersecting and intersecting equilibrium beach profiles resulting from beach nourishment. With sand of different characteristics, the method can accommodate varying ranges of realism, from idealized initial profile and grain size distribution nourished profiles represented by analytical forms, to the most realistic case, in which the initial profile and nourishment grain size distribution are arbitrary and are user-specified. The equilibrium-nourished profile is based locally on the differential equation for equilibrium for a distribution of sizes. Examples are presented illustrating the influence of various parameters.

4 Laboratory Studies

Introduction and Description of Facilities

An exploratory series of laboratory studies were conducted to investigate profile response to waves of different characteristics for the case of a poorly sorted sediment. Six experiments were conducted using the same sand, but with different initial beach slopes and wave conditions.

The test conditions for the laboratory experiments are presented in Table 1, and a schematic of the facilities is shown in Figure 18.

Table 1
Experimental Conditions, Wave Tank Tests

Run No.	Wave Period T(sec)	Wave Height H(cm)	Water Depth h(cm)	Initial Slope
1	1.25	9.0	22.5	1:10.85
2	1.25	9.0	22.0	1:5.74
3	1.25	11.0	22.5	1:9.93
4	1.25	11.0	21.0	1:13.94
5	1.25	8.5	20.0	1:24.27
6	1.25	9.0	21.0	1:14.33

Objectives

The general objectives of the laboratory studies were:

a. To document the evolution and sorting with time of initially planar beach profiles and poorly sorted sediments.

b. To compare experimental profiles and sediment size distributions with predicted values based on techniques developed in this study.

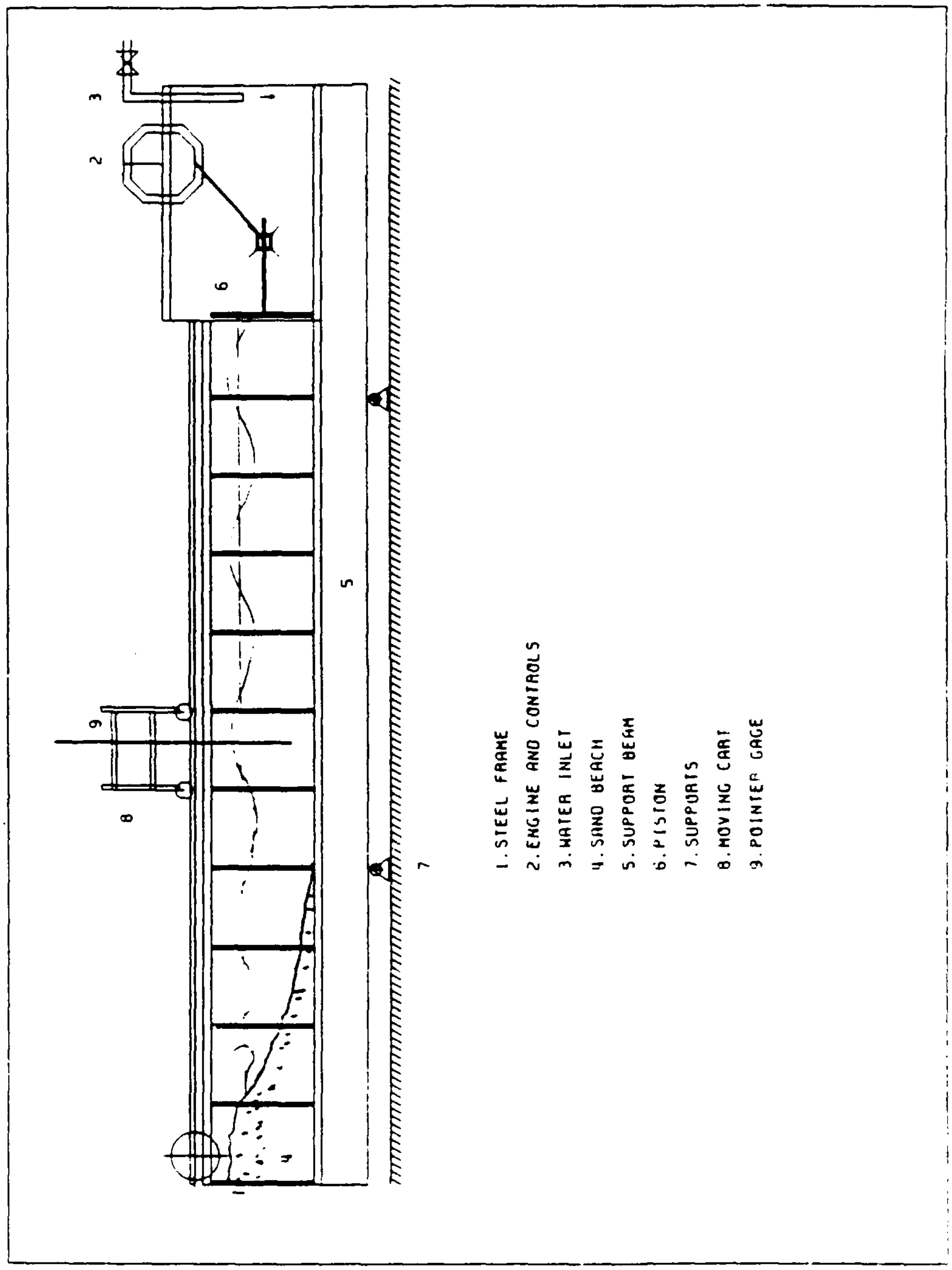


Figure 18. Schematic of laboratory facilities

Experiment Procedures

The six tests commenced with the initially planar beach slopes presented in Table 1. Prior to establishing the initial profile, the sand from across the profile was mixed to approximate uniformity. After establishing the uniform slope, sand samples at four or five different locations across the profile were collected for later analysis.

The desired wave conditions were established and the profile documented and sand sampling repeated at 1, 5, 10, and 24 hr (Experiments 1-3) and 1, 6, 12, and 24 hr (Experiments 4-6). The wave heights were measured visually and the location and height of the breaking waves were documented several times during the test.

Test Results

The results of the test program are described below for each of the experiments.

Experiment 1

The profile evolution from a planar slope of 1:10.85 is presented in Figure 19 for the initial profile and profiles at 1, 5, 10, and 24 hr of testing. It is seen that only minor changes occurred between 10 and 24 hr, indicating that the system had approached equilibrium. The general characteristics of the final profile relative to the initial include a concave upward profile with most of the sand transported seaward and only a minor amount transported landward to form a berm feature. Figure 20 presents the variation with time of mean grain diameter for four locations across the beach for all six experiments at 0, 10, and 24 hr. For Experiment 1 (Figure 20a), it is clear that the initial mean grain size was reasonably uniform across the beach and that, with progressing time, the coarser sediments were transported shoreward and the finer sediments seaward. Figure 21 presents the initial and final profiles and the grain size distributions at each of three locations across the profile, and the initial grain size distribution. Figure 22 shows the grain size distribution at 24 hr at six locations across the profile. For this experiment, substantial cross-profile sorting is evident, with the coarser sediment concentrated near the water line.

Experiment 2

The evolution of this profile is presented in Figure 23 for times of 0, 1, 5, 10, and 24 hr. This profile commenced with a relatively steep uniform slope (1:5.74) and practically all of the sediment transport was seaward. Variations with time of mean diameters at four locations across the profile are in Figure 20b. There is substantially less pattern to the mean size distributions compared to Experiment 1. There has been some reduction

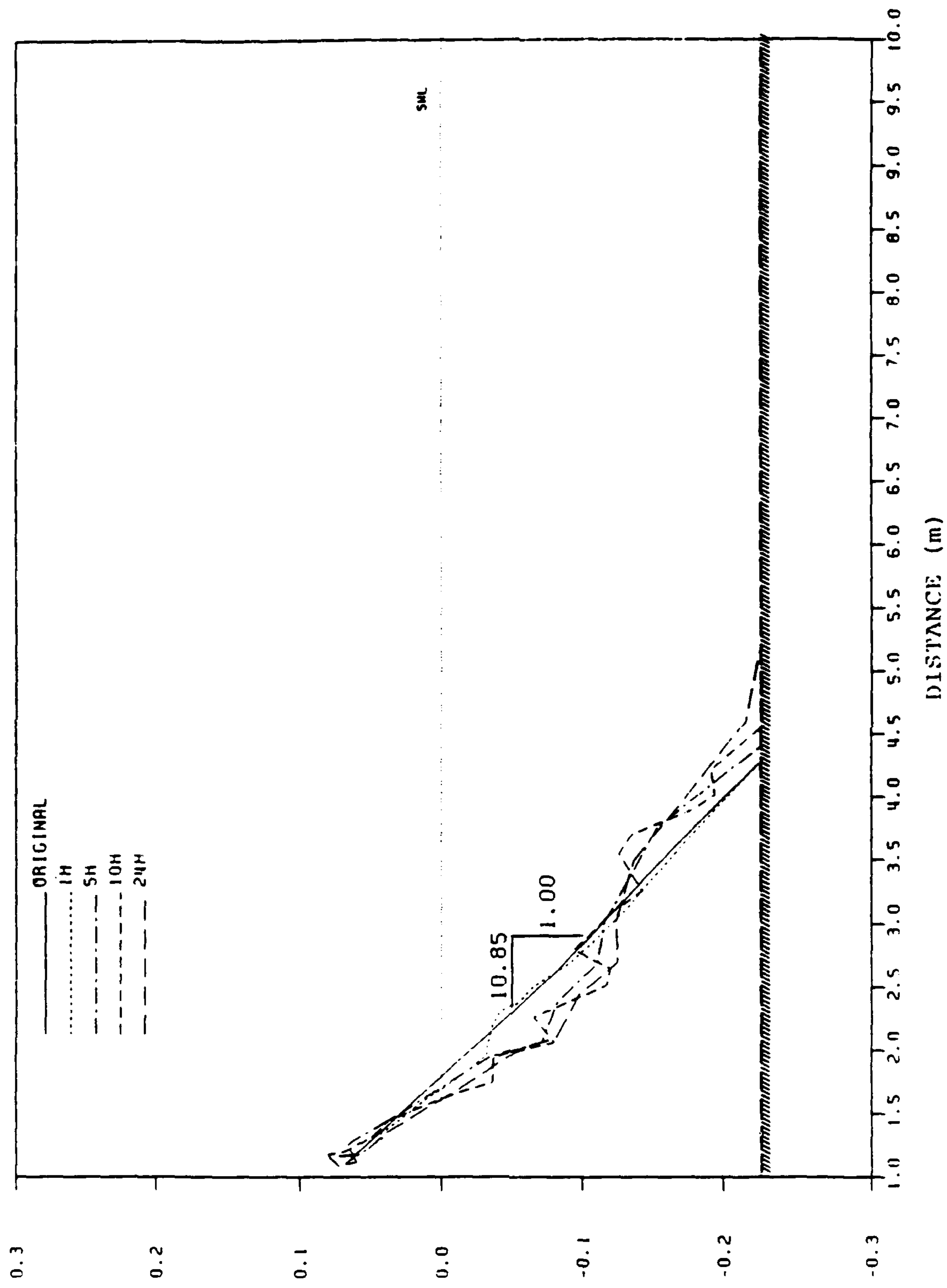


Figure 19. Experiment 1, measured profiles at various times

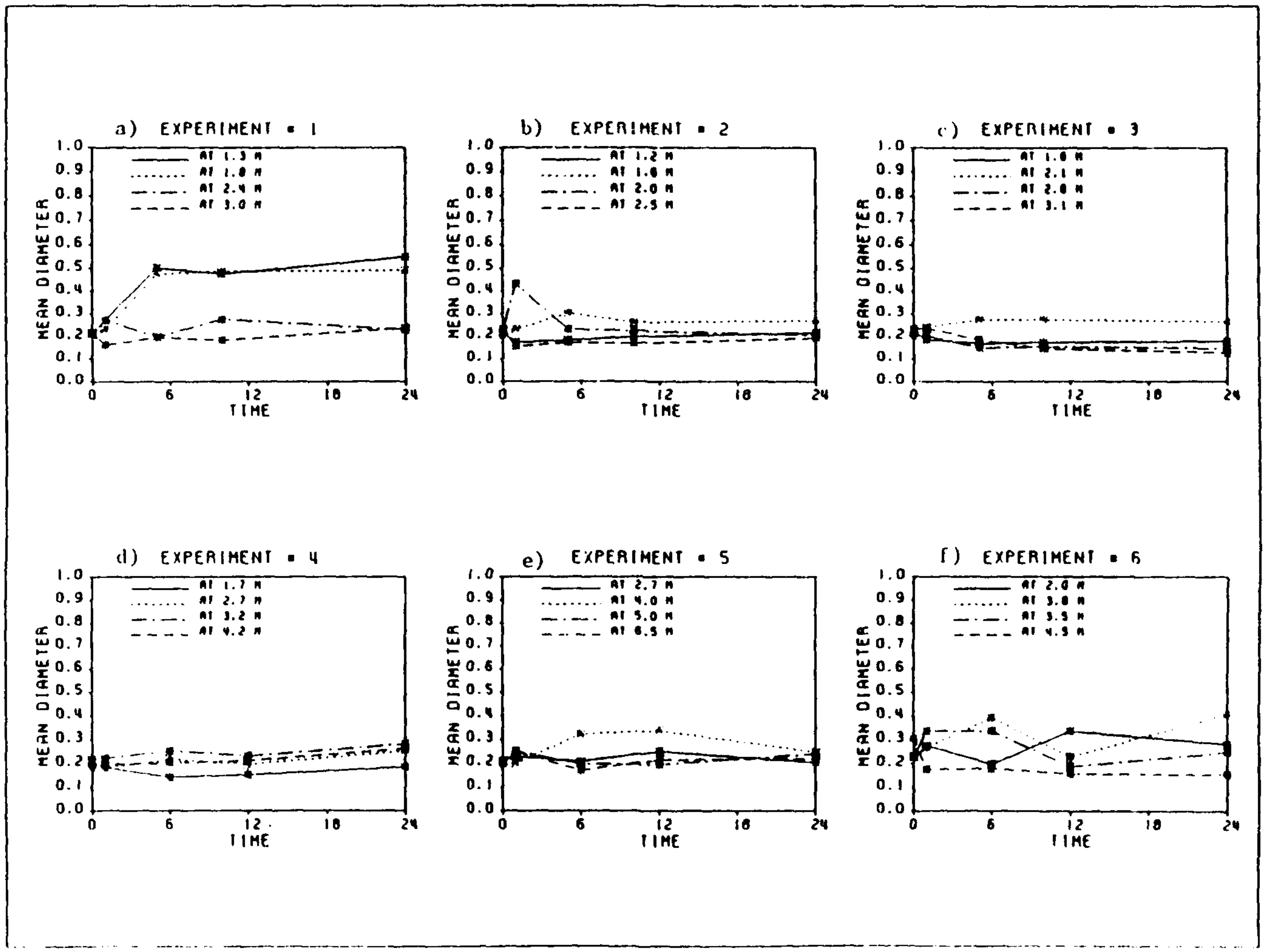


Figure 20. Variation of mean sediment size with time at several locations across the beach, all six experiments. Sediment diameter in millimeters

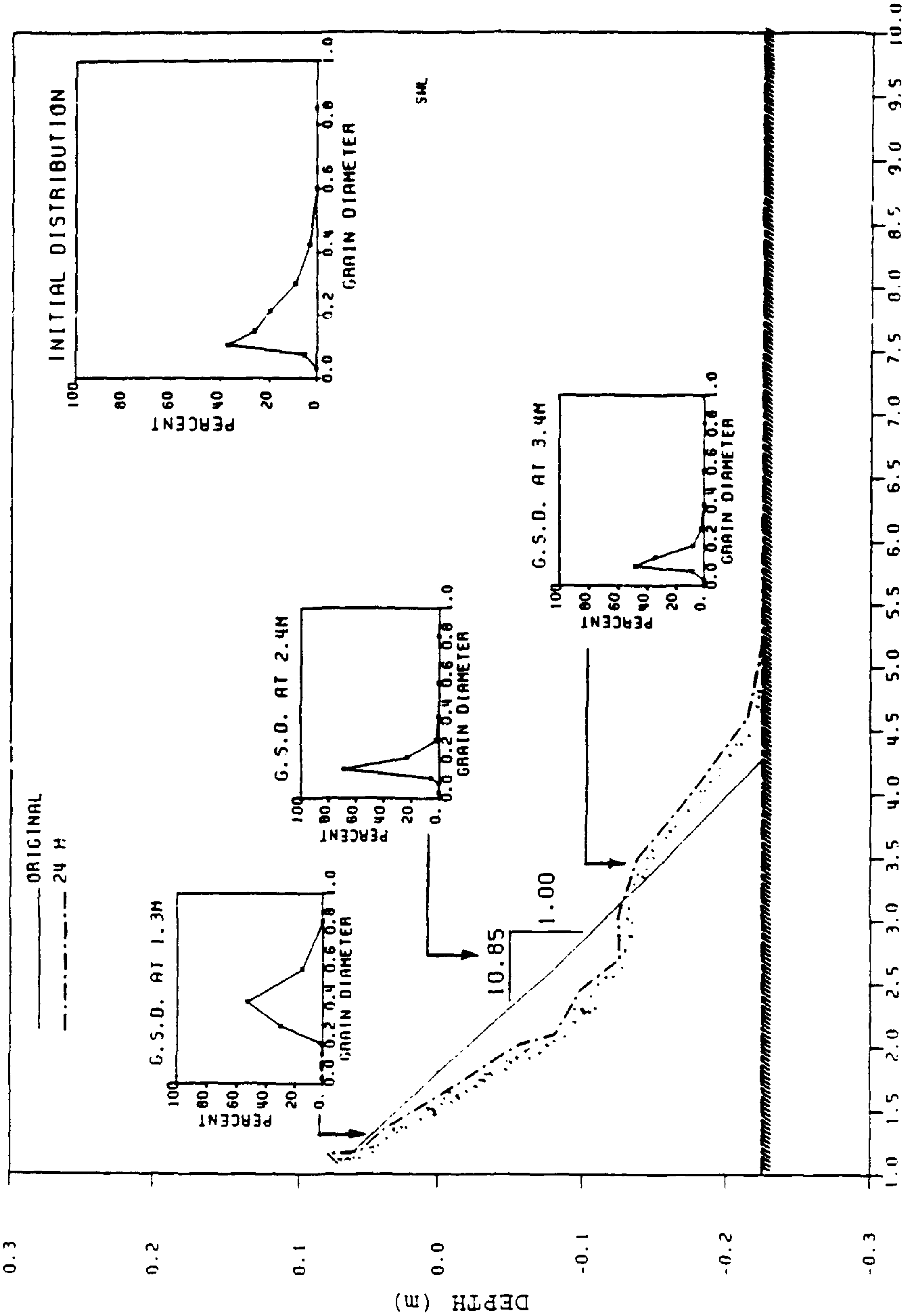


Figure 21. Experiment 1, initial and final profiles and initial and final grain size distributions

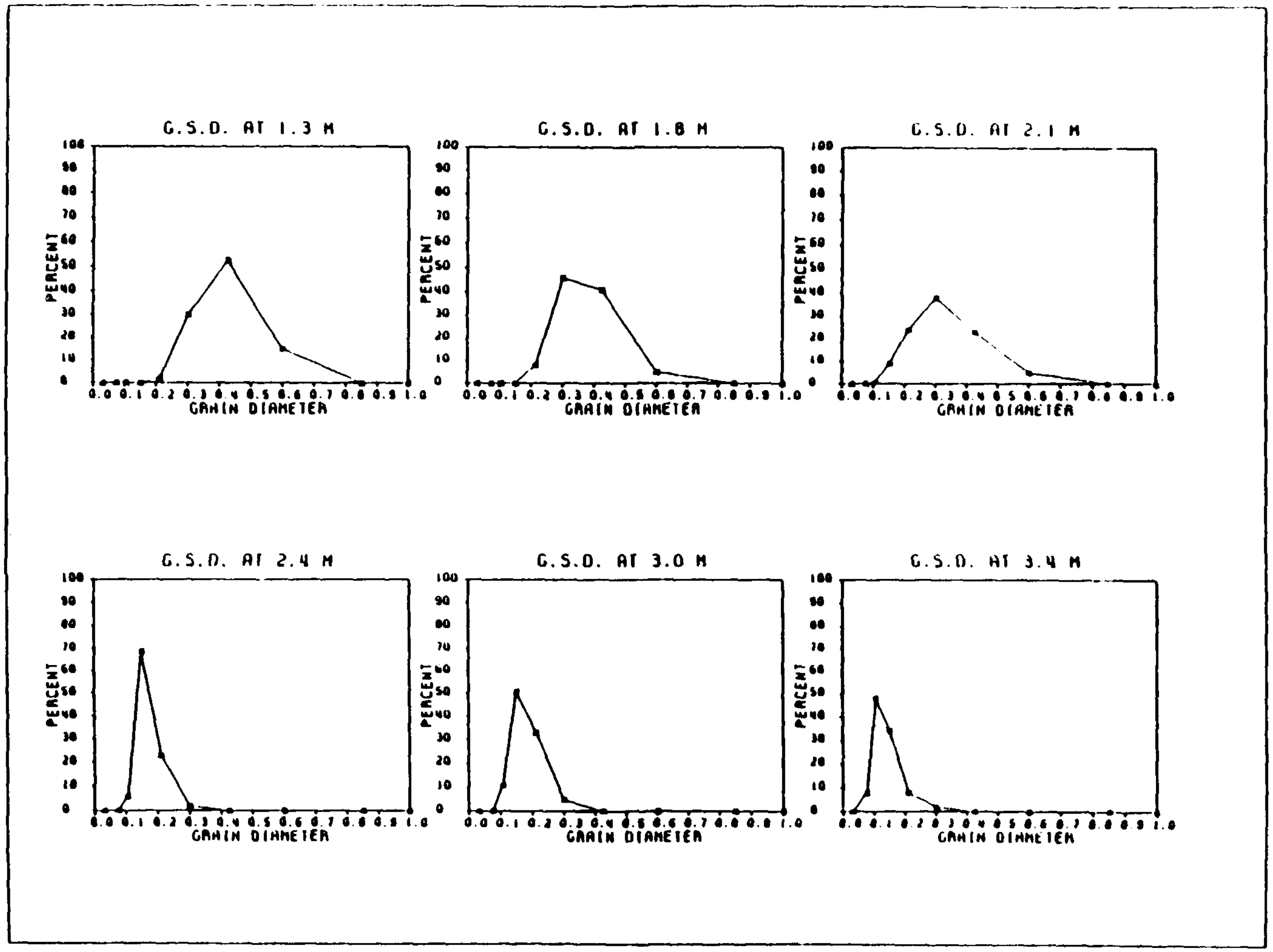


Figure 22. Experiment 1, grain size distributions at six locations across the profile after 24 hr of testing

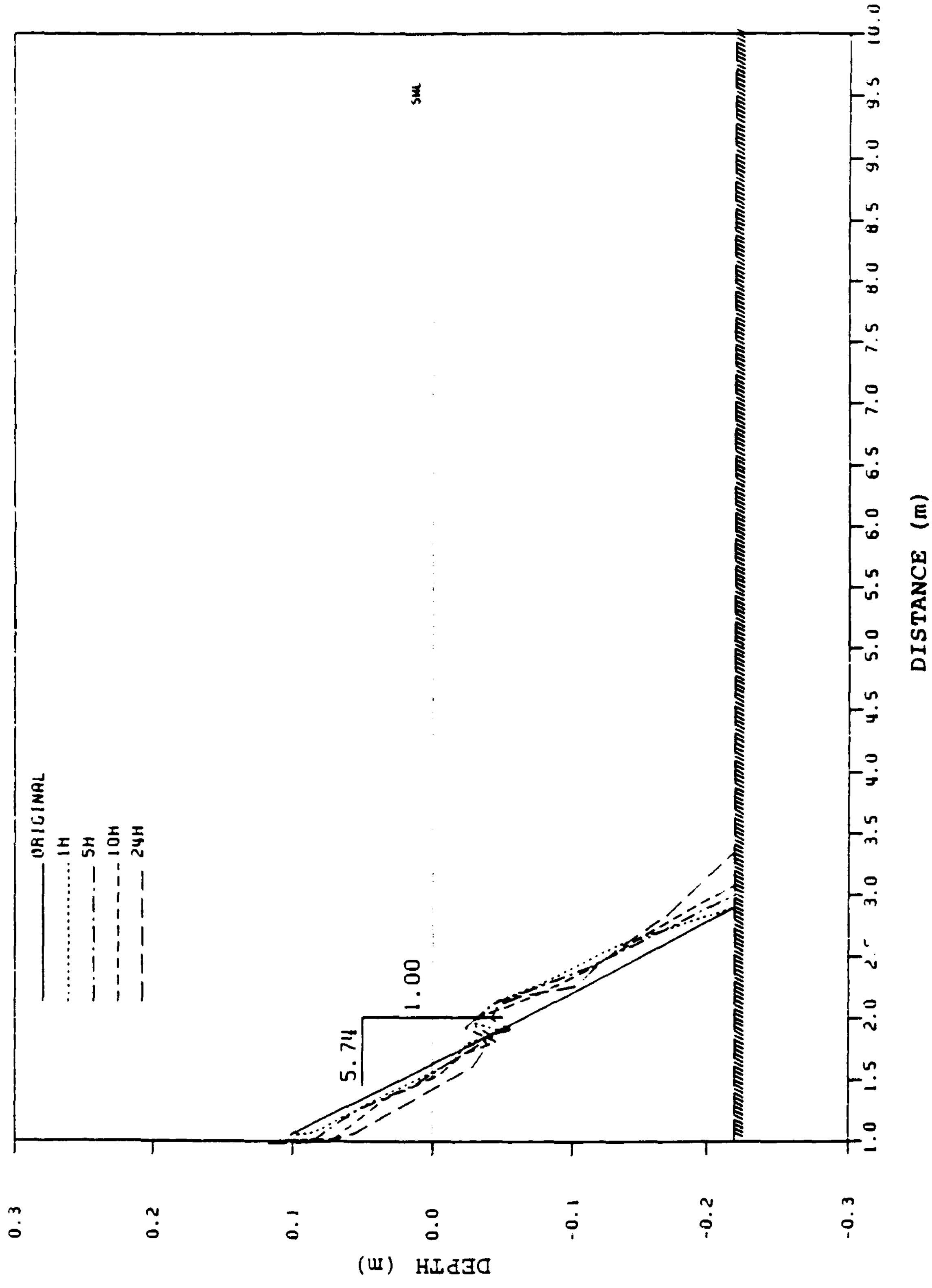


Figure 23. Experiment 2, measured profiles at various times

in mean grain size in the seaward portion of the profile, and some size increases toward shore, with very little change at the most shoreward location sampled. The initial and final profiles and the final grain size distributions at three locations across the profile are presented in Figure 24, along with the initial grain size distribution. Figure 25 presents the grain size distribution at six locations across the profile after 24 hr of testing.

Experiment 3

Figure 26 presents the profile evolution for Experiment 3 at 0, 1, 5, 10, and 24 hr. This result is of interest in that wave and initial slope conditions are nearly the same as presented in Experiment 1, except that most of the sediment transport was shoreward in Experiment 3. Variations with time of mean grain sizes at four locations across the beach at 0, 5, 10, and 24 hr are presented in Figure 20c. Figure 27 presents the initial and final profiles and the grain size distributions at three locations across the profile. Grain size distributions at six locations across the profile at the final (24 hr) survey are presented in Figure 28. In general, sorting across the profile has occurred.

Experiment 4

Some improvements to the experiment arrangement were made before running this experiment, including water level control and recording of the waves. The conditions involved a slope of 1:13.94, which is milder than in the previous experiments. The other variables were kept in the same range. As in the former experiment, irregular waves occurred.

Inspection of Figure 29 shows that a berm and a bar were formed as in the first experiment. The volumes do not match as well as in previous cases, which is due to lack of lateral symmetry and the substantial consolidation observed at the very beginning of the experiment. Nevertheless, erosion at the offshore end of the beach was observed. Figure 29 shows the profiles for 0, 1, 6, 12, and 24 hr. Grain size variations are shown in Figure 30. The variations with time of mean grain sizes at four locations across the profile are presented in Figure 20d. Figure 31 provides the grain size distributions at six locations across the profile at the final (24-hr) survey.

Although a clear trend in grain sizes is not evident, it can be seen that the most seaward sample has about the same distribution as the original and that, this time, a sorting to finer sizes has been achieved at the berm.

Experiment 5

This experiment included the mildest initial slope of all six tests. A berm trapping a lagoon was formed with the sand apparently originating from both onshore and offshore sides of the beach. Also, an offshore bar was formed clearly with sand provided by the zone in between. The evolution of the

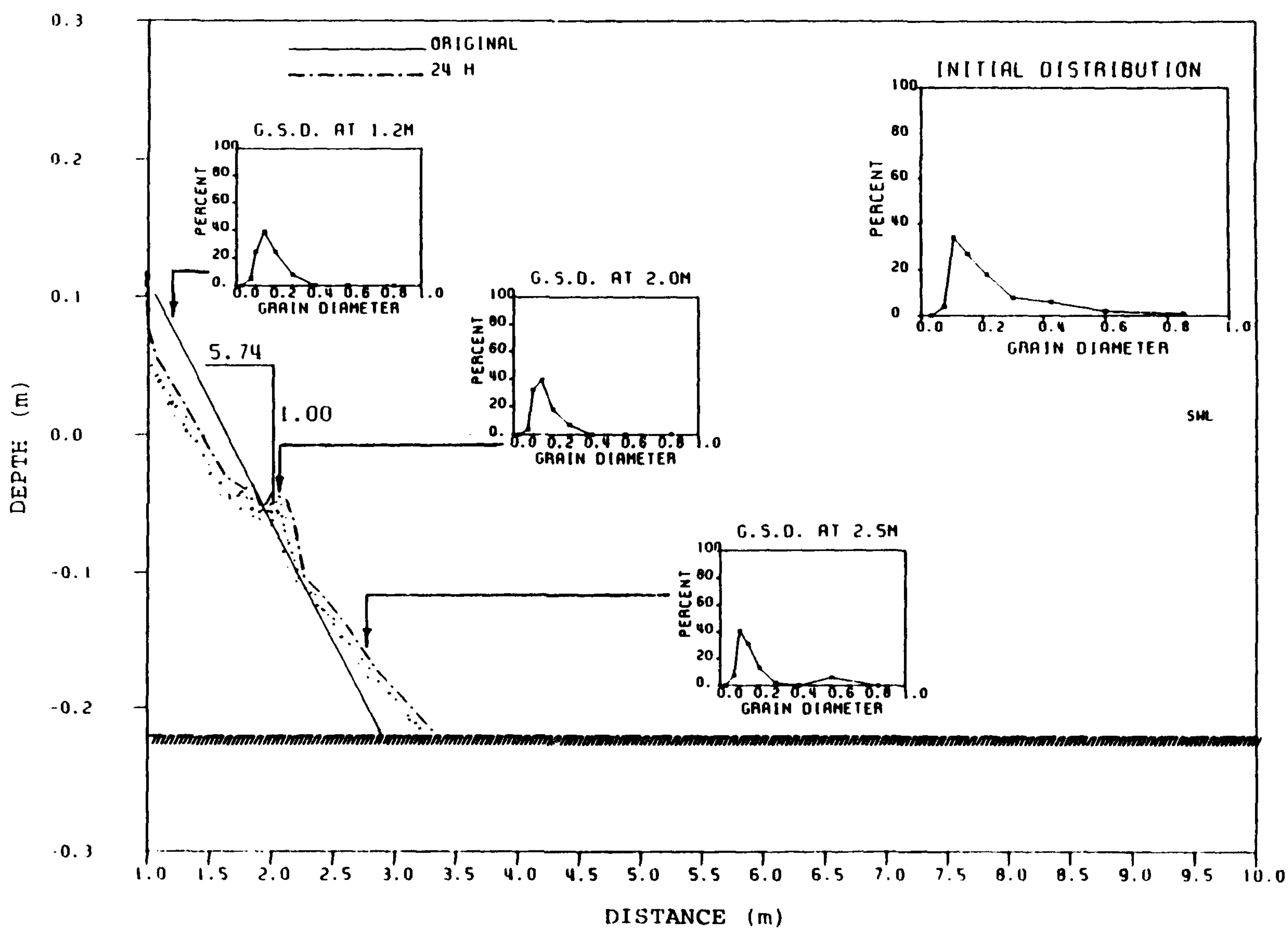


Figure 24. Experiment 2, initial and final profiles and initial and final grain size distributions

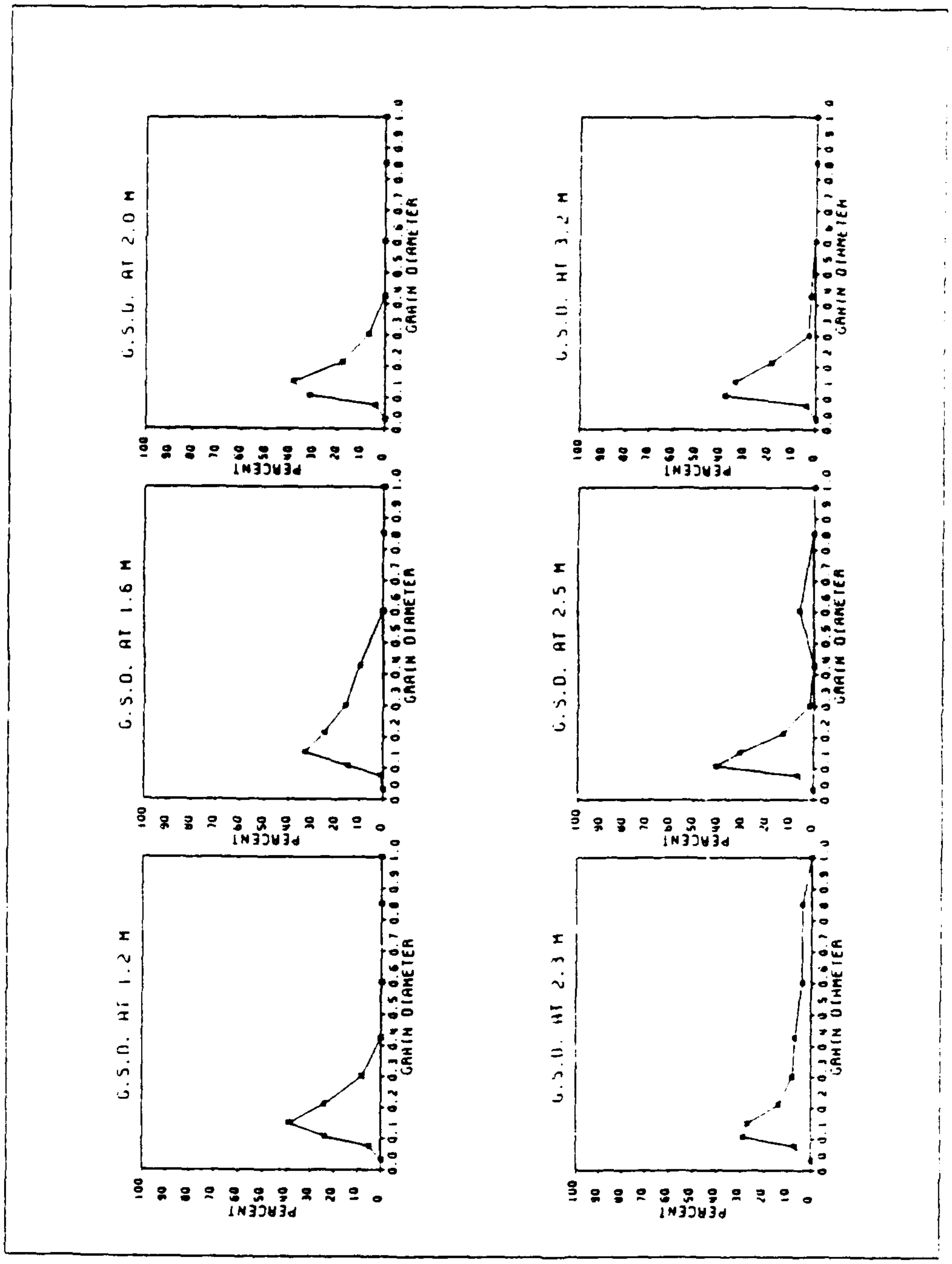


Figure 25. Experiment 2, grain size distributions at six locations across the profile after 24 hr of testing

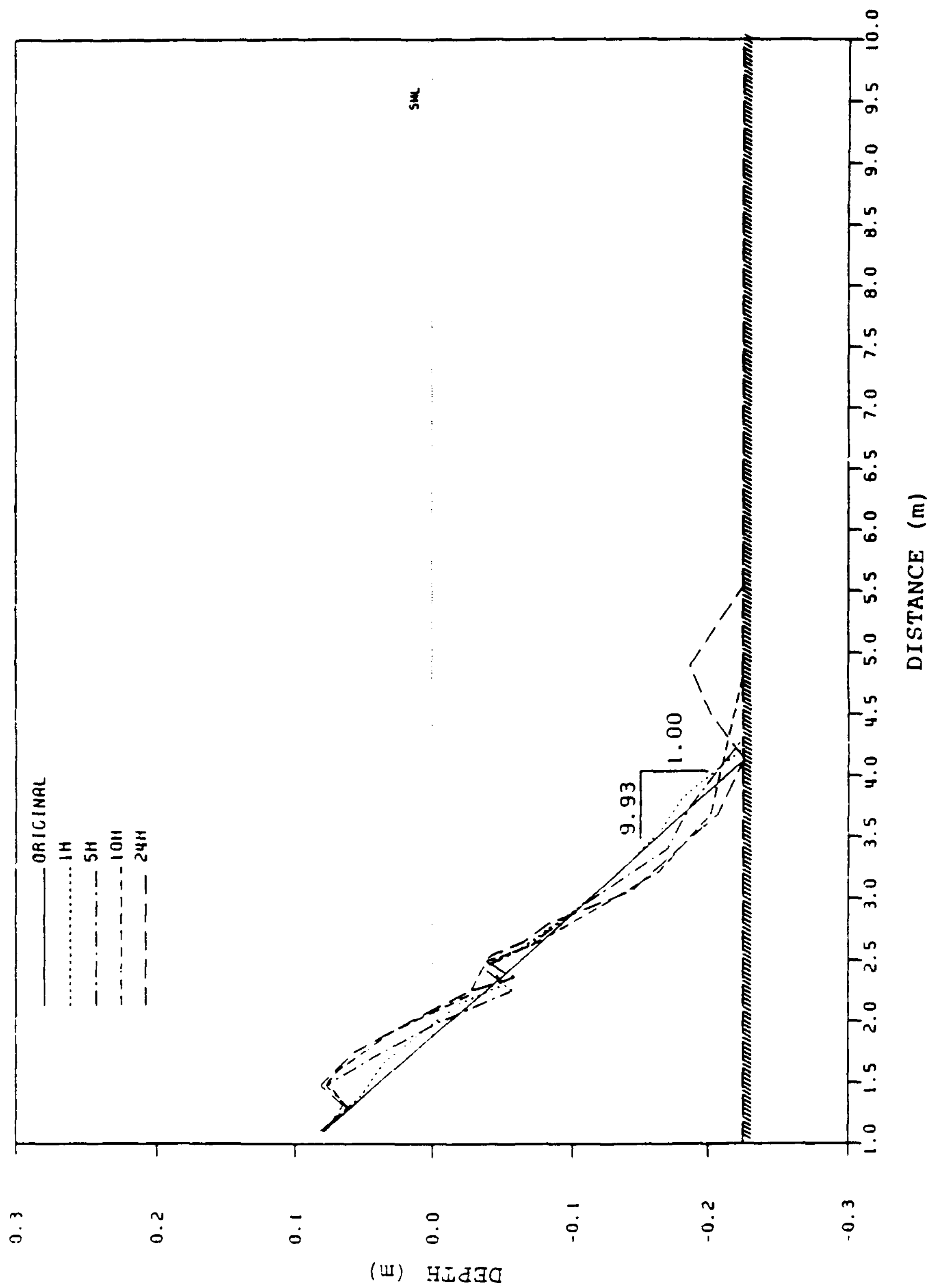


Figure 26. Experiment 3, measured profiles at various times

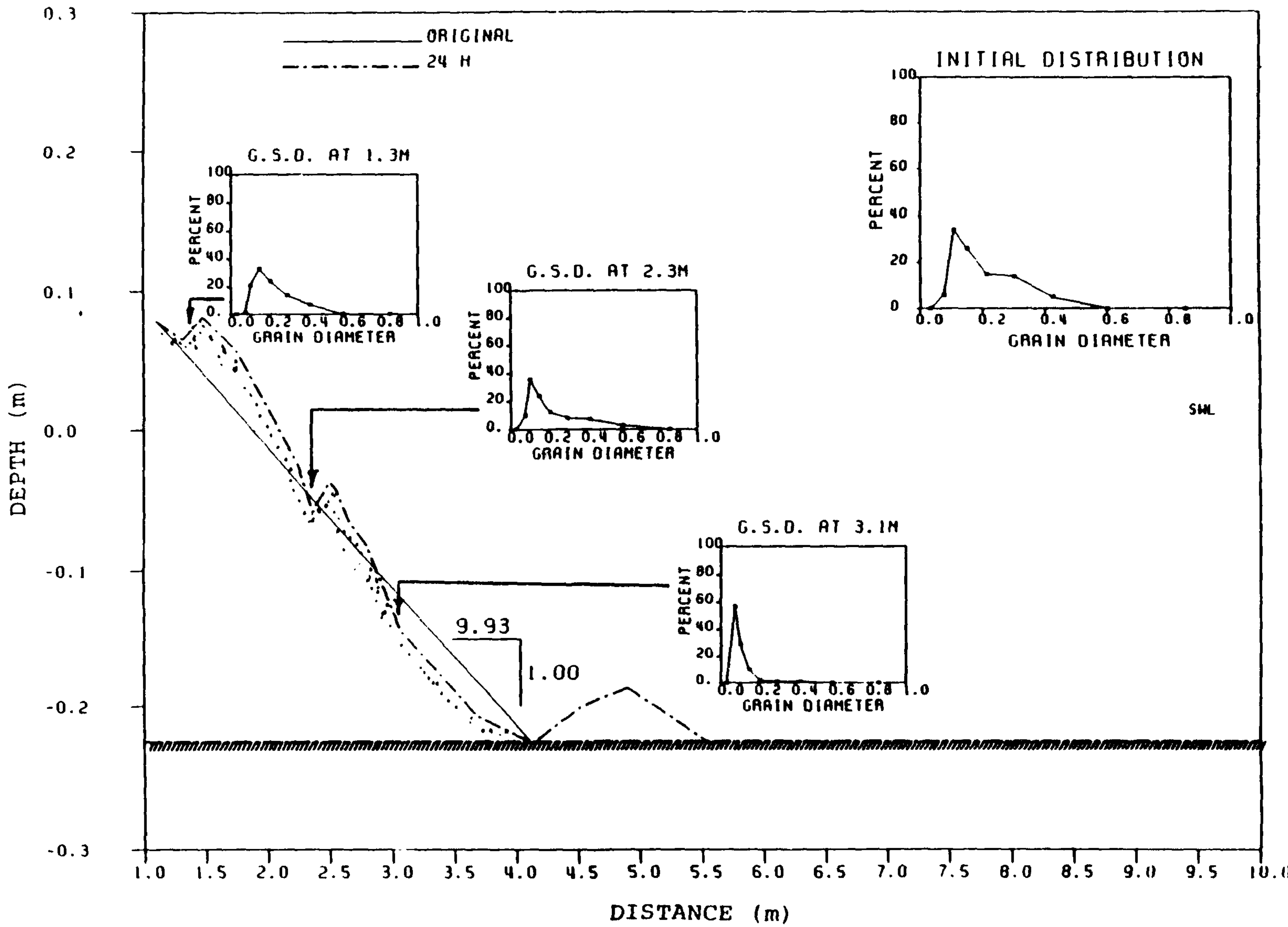


Figure 27. Experiment 3, initial and final profiles and initial and final grain size distribution

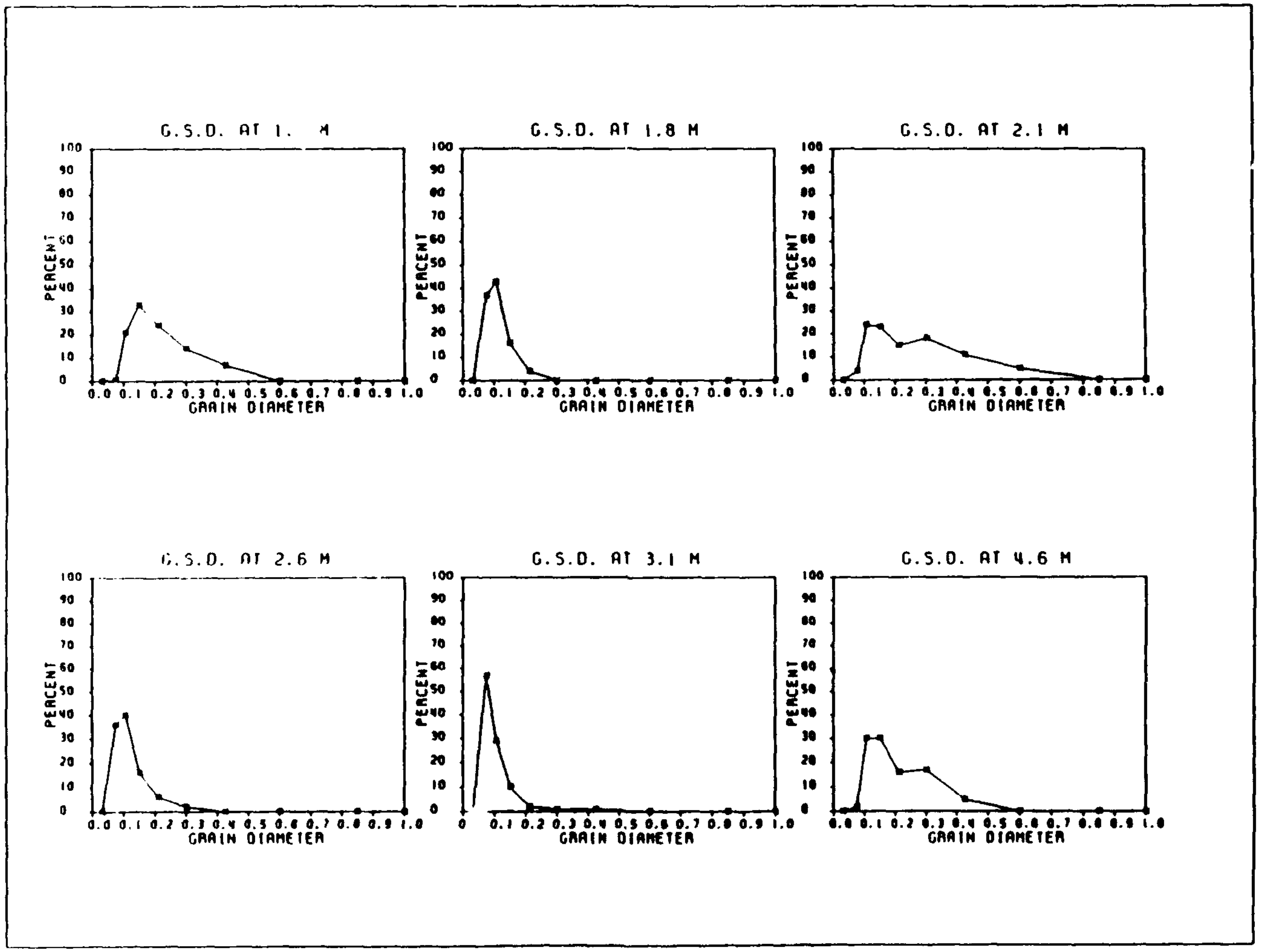


Figure 28. Experiment 3, grain size distributions at six locations across the profile after 24 hr of testing

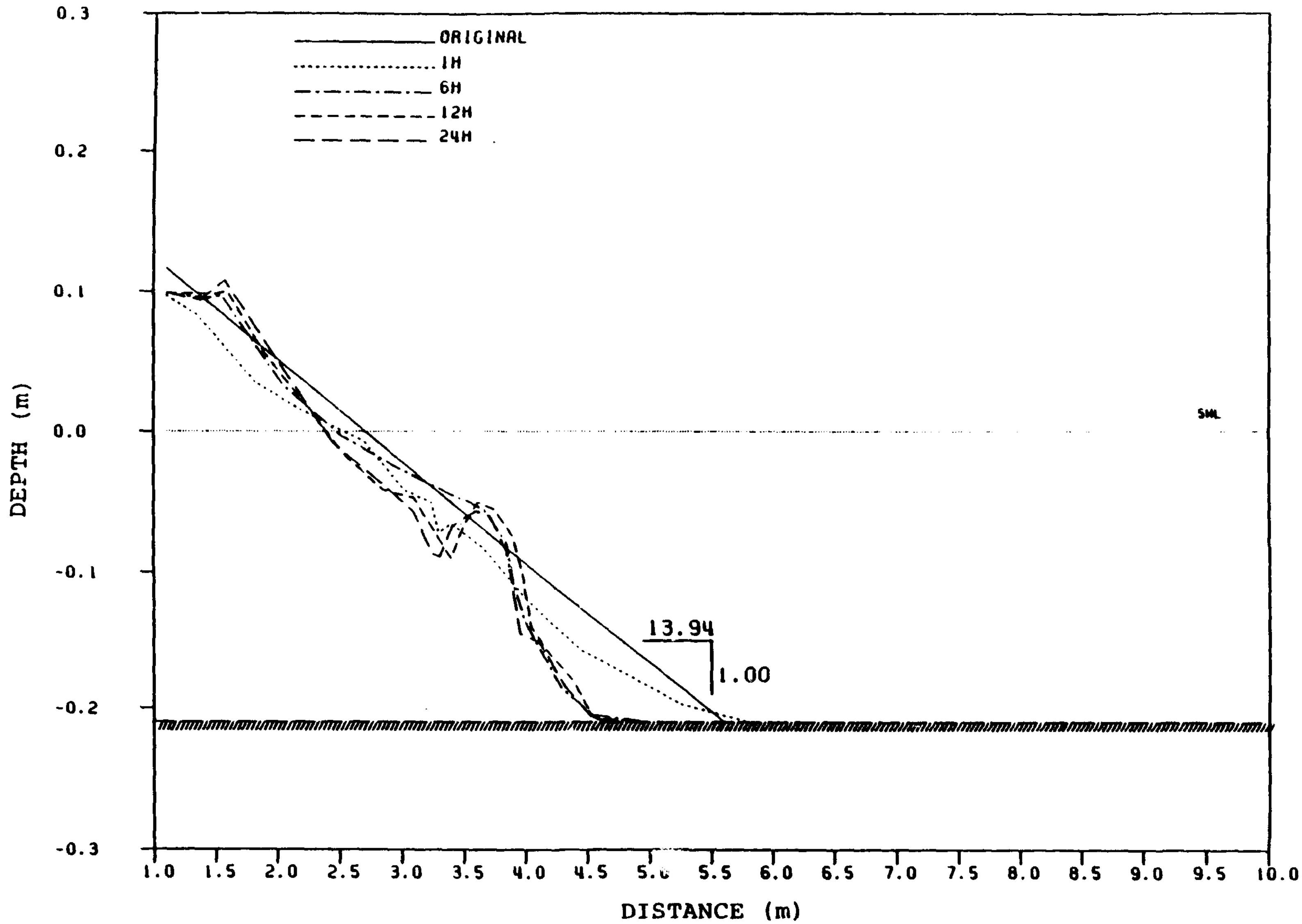


Figure 29. Experiment 4, measured profiles at various times

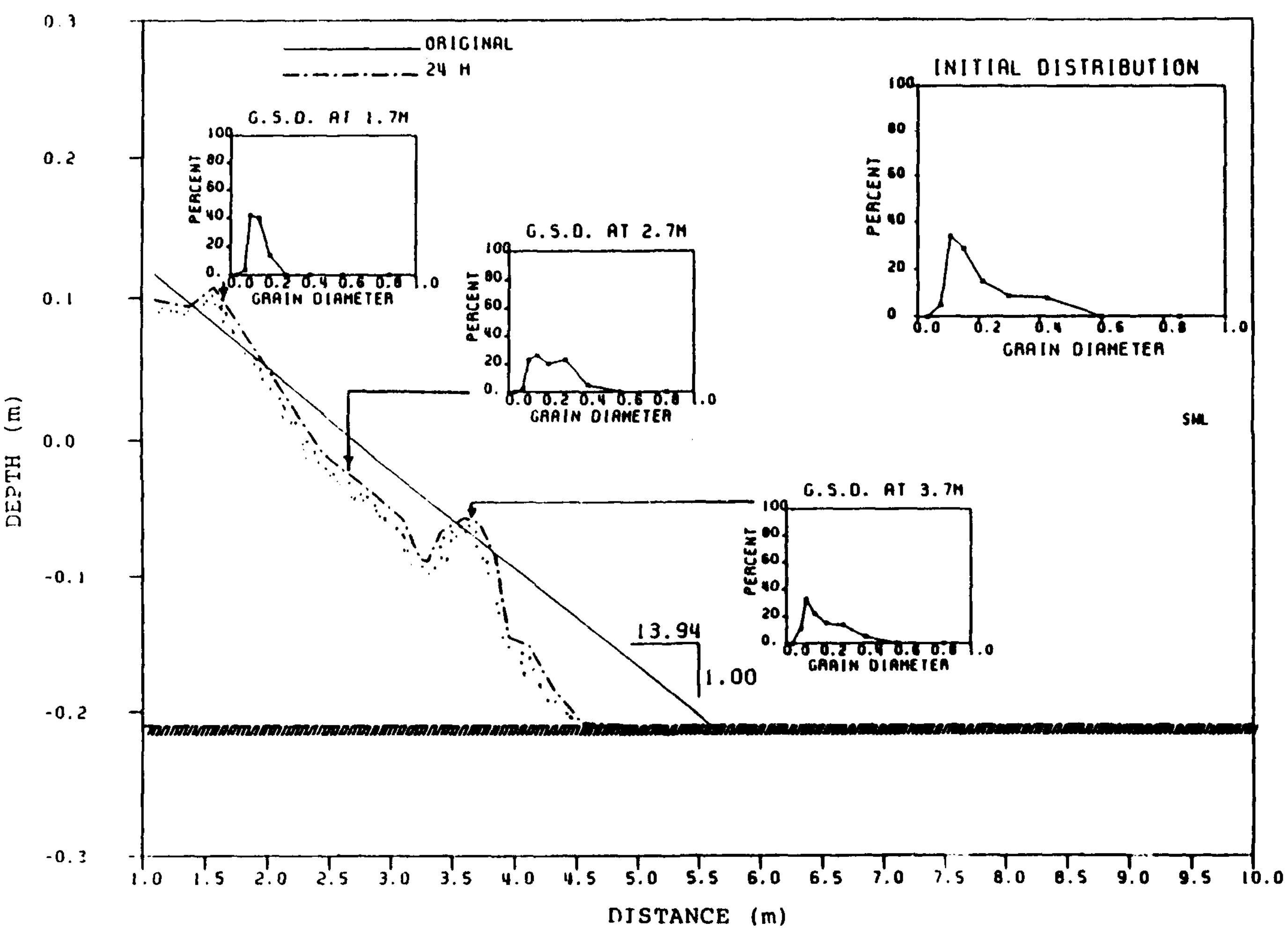


Figure 30. Experiment 4, initial and final profiles and initial and final grain size distributions

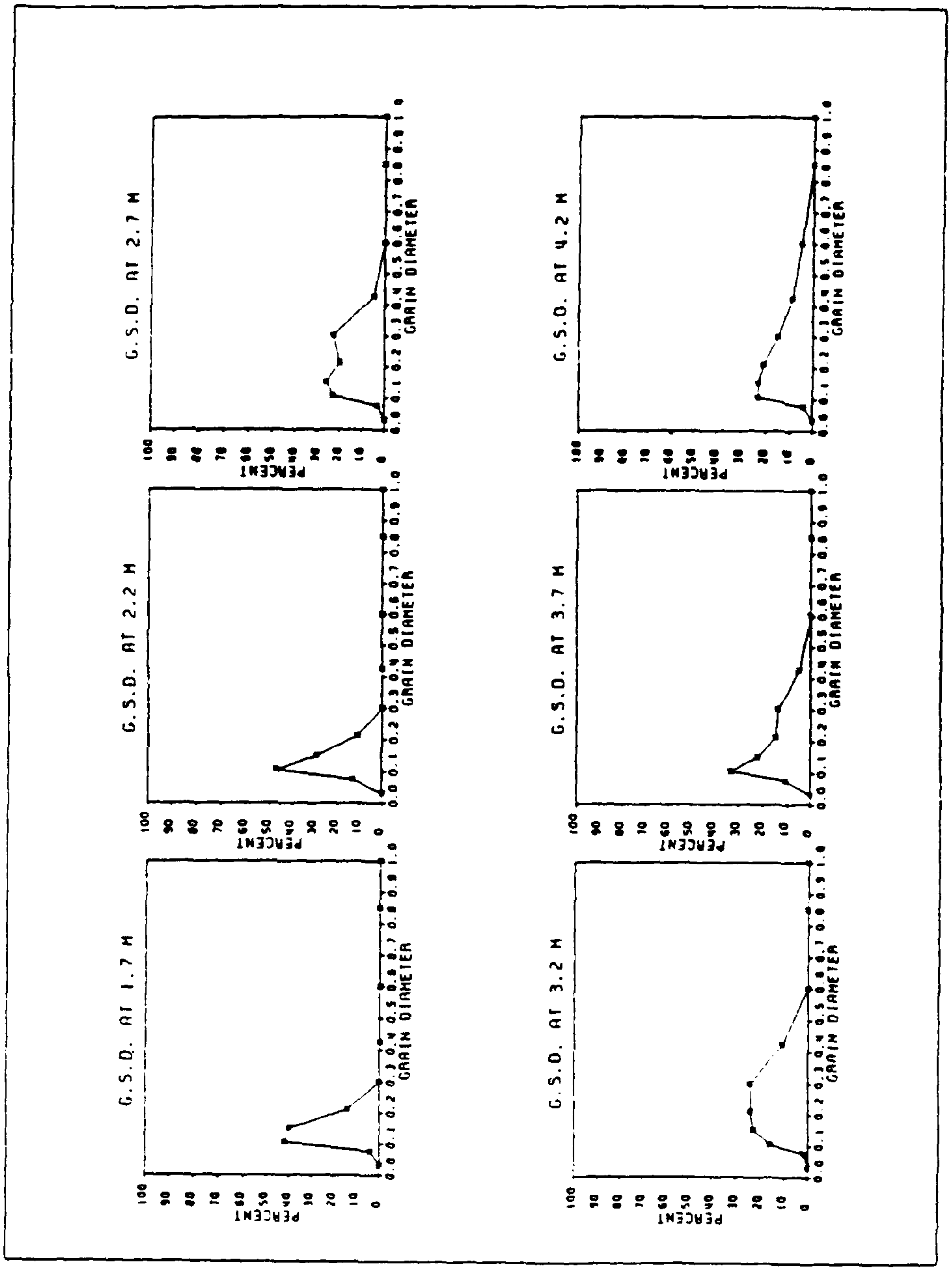


Figure 31. Experiment 4, grain size distributions at six locations across the profile after 24 hr

beach is shown in Figure 32 for 0, 1, 6, 12, and 24 hr. The initially mild slope became even milder on the average below the still-water level. From Figure 33, which shows the grain size distributions for the initial and final, it can be concluded that the concentration of fine grain sizes offshore is greater. The variation with time of mean grain sizes at four locations across the profile is presented as Figure 20e. Figure 34 provides the grain size distributions at six locations across the profile at the final (24-hr) survey.

Experiment 6

This was the most sophisticated experiment because of the use of sand tracers. The evolution of the beach is shown in Figure 35. As in Experiment 1, a berm was formed but most of the sand was transported offshore where it formed a bar feature.

The sand size distributions for the initial and final profiles are shown in Figure 36. It is clear that far offshore the sand is finer even though from this figure the location of the coarsest sand is not so evident. Some shift to coarser sizes from the initial occurs at the berm and at 3.0 m.

The tracers were followed visually as carefully as possible, but this was not always an easy task.

The initial tracer sieve sizes and distributions were as follows:

a. Blue: #100 at 4.5 m.

b. Orange: #100 at shoreline.

c. Magenta: #70 at 4.5 m.

d. *Green: #70 at shoreline.*

e. Yellow: #50 at 4.5 m.

f. Red: #50 at shoreline.

The results of the tracer investigation are summarized in Table 2. The following can be concluded from the analysis:

a. The red tracers were the most readily tracked.

b. The blue tracer was lost completely. This was probably due to the small size of these grains (#100) and their initial location in relatively deep water. The interpretation is that when the action of the waves began they were suspended and spread over a large area with a very low concentration, making it difficult to follow their path or even to find them in a later examination of the sand.

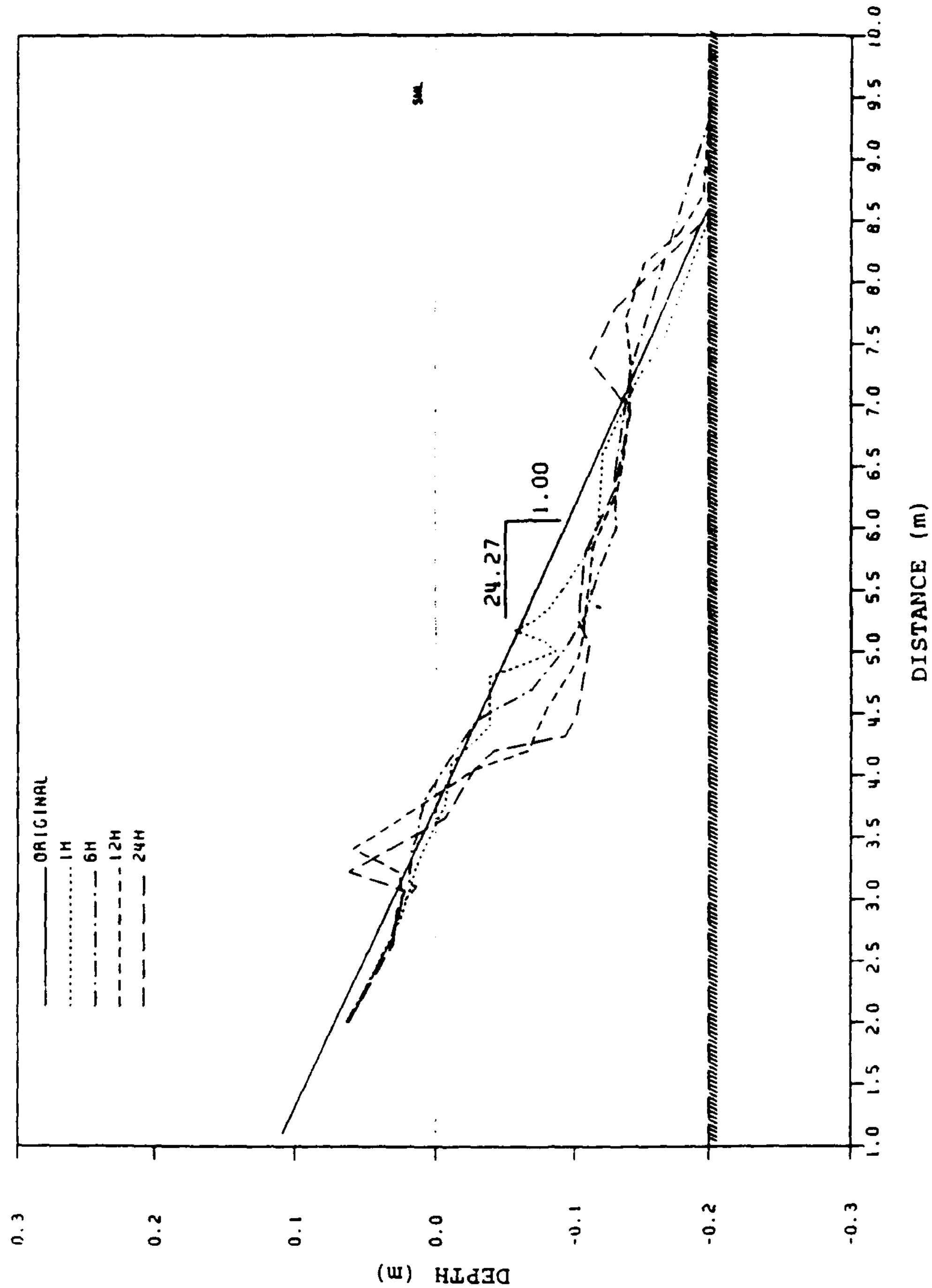


Figure 32. Experiment 5, measured profiles at various times

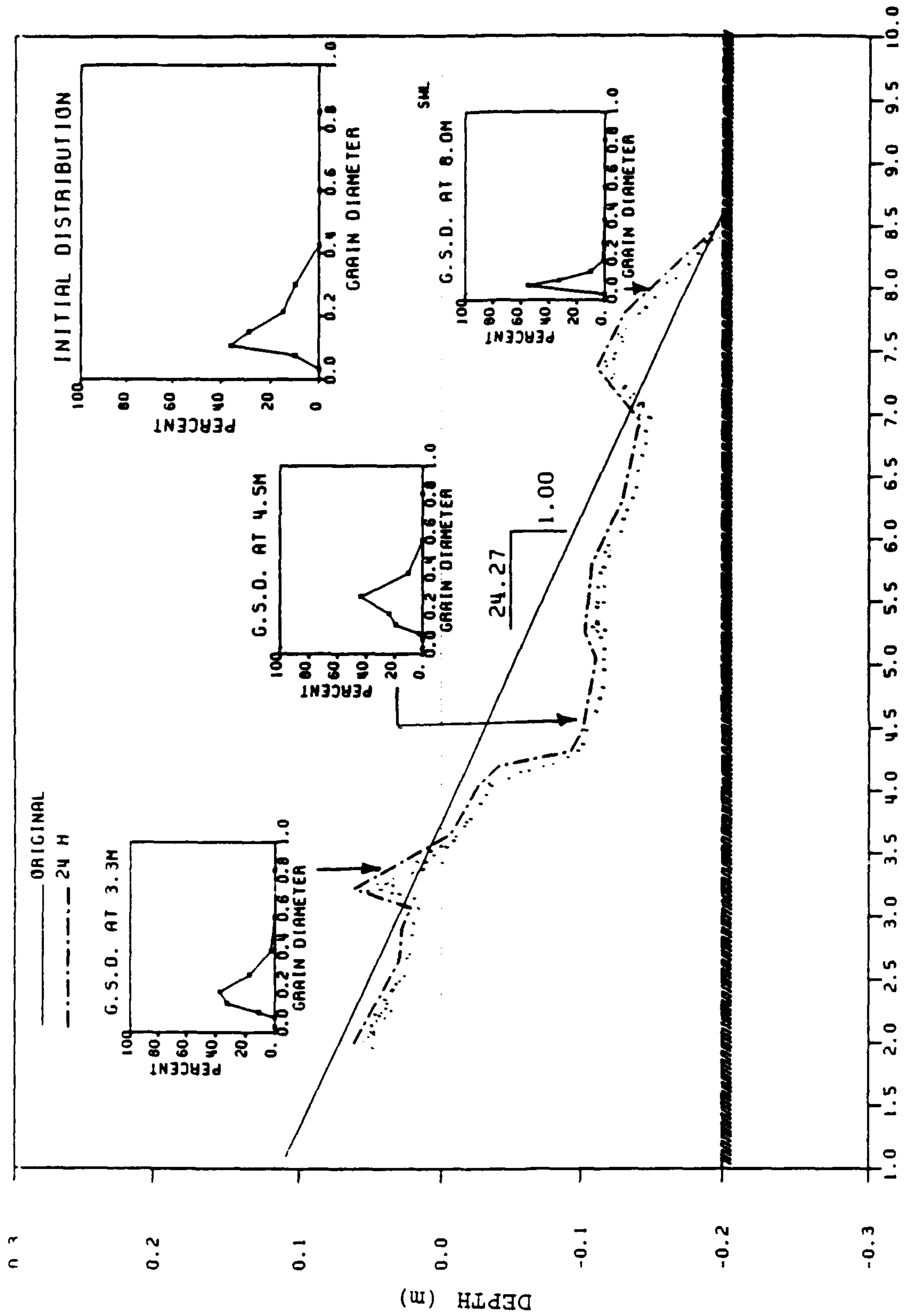


Figure 33. Experiment 5, initial and final profiles and initial and final grain size distributions

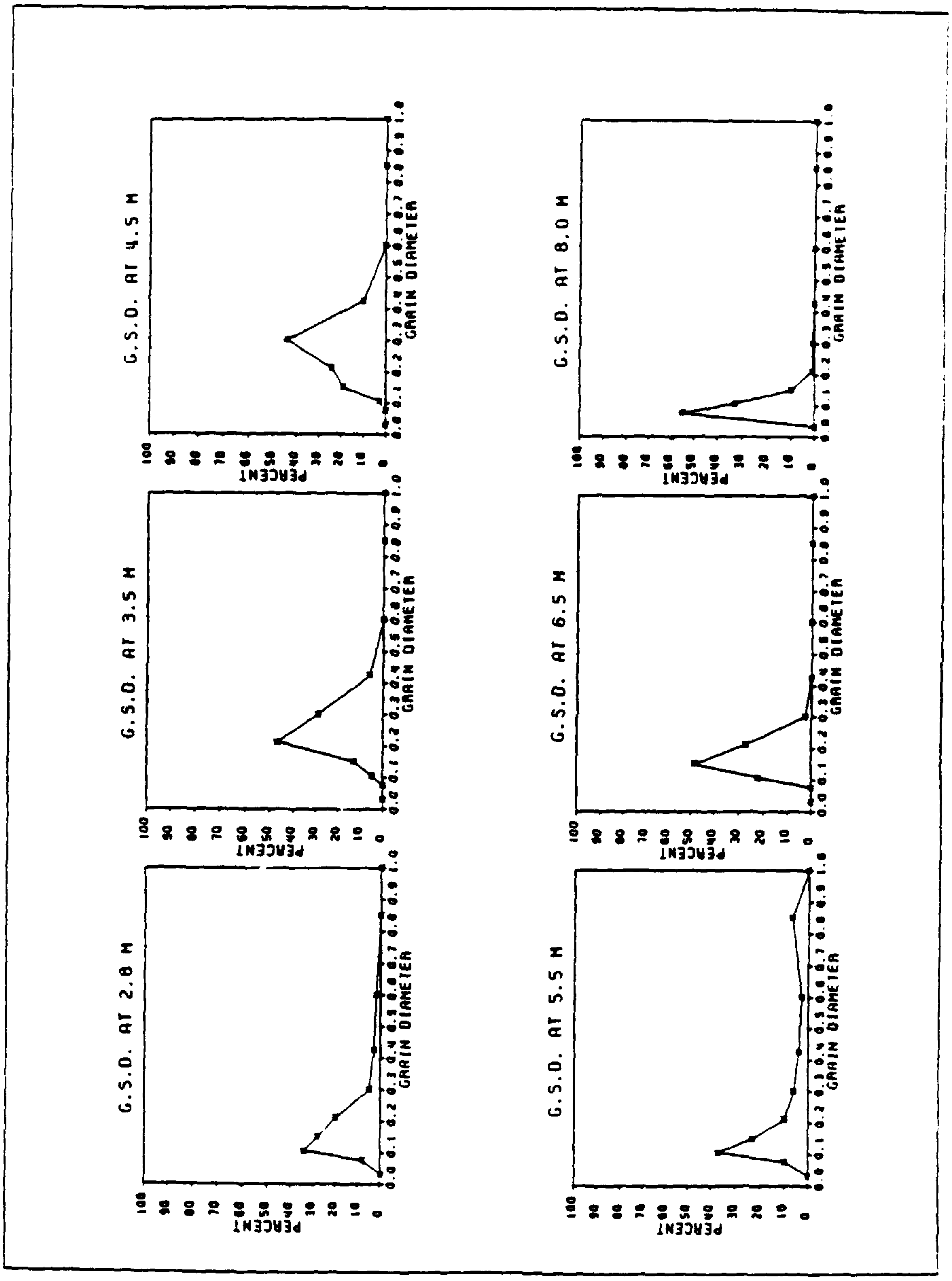


Figure 34. Experiment 5, grain size distributions at six locations across the profile after 24 hr

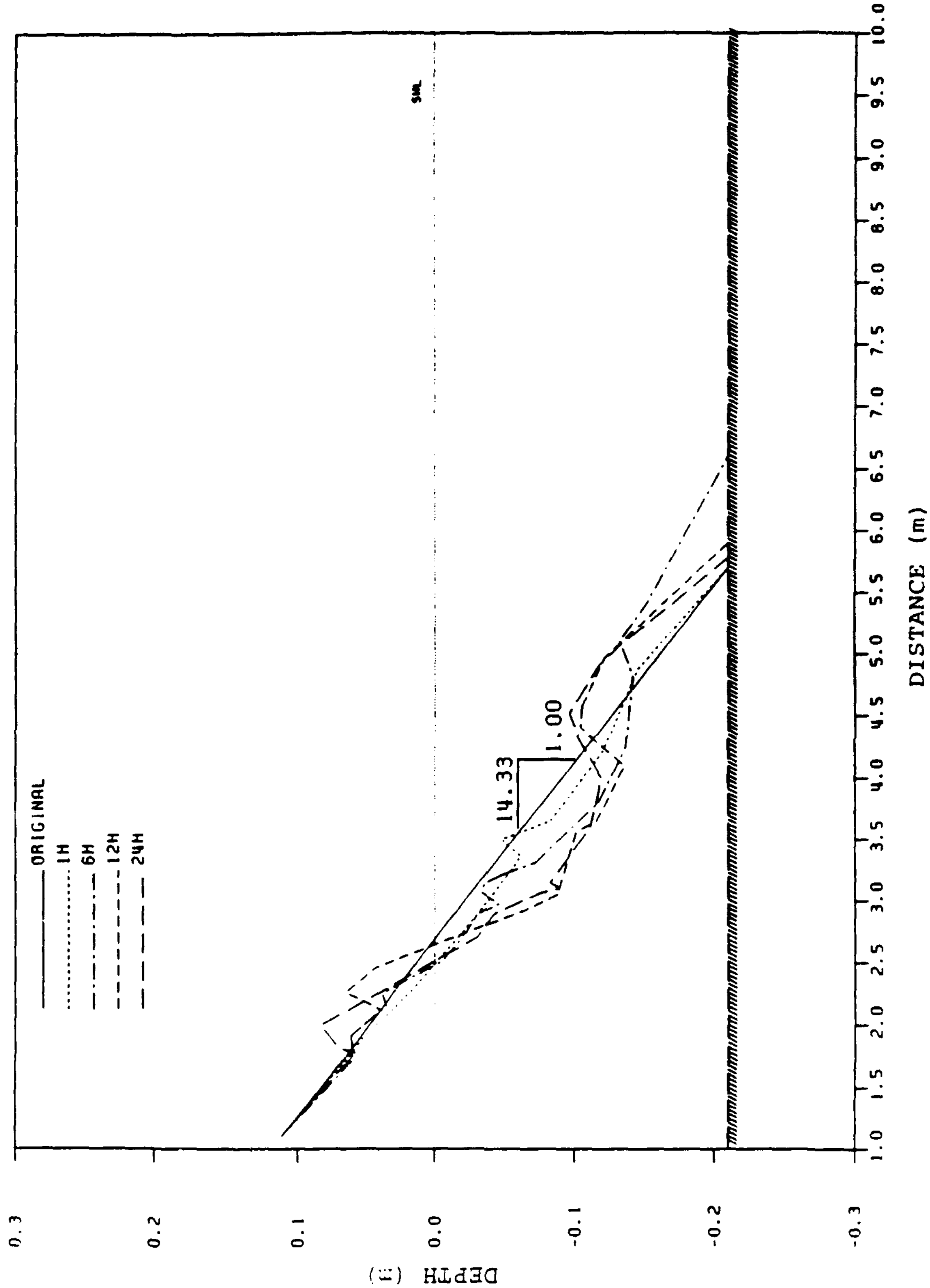


Figure 35. Experiment 6, measured profiles at various times

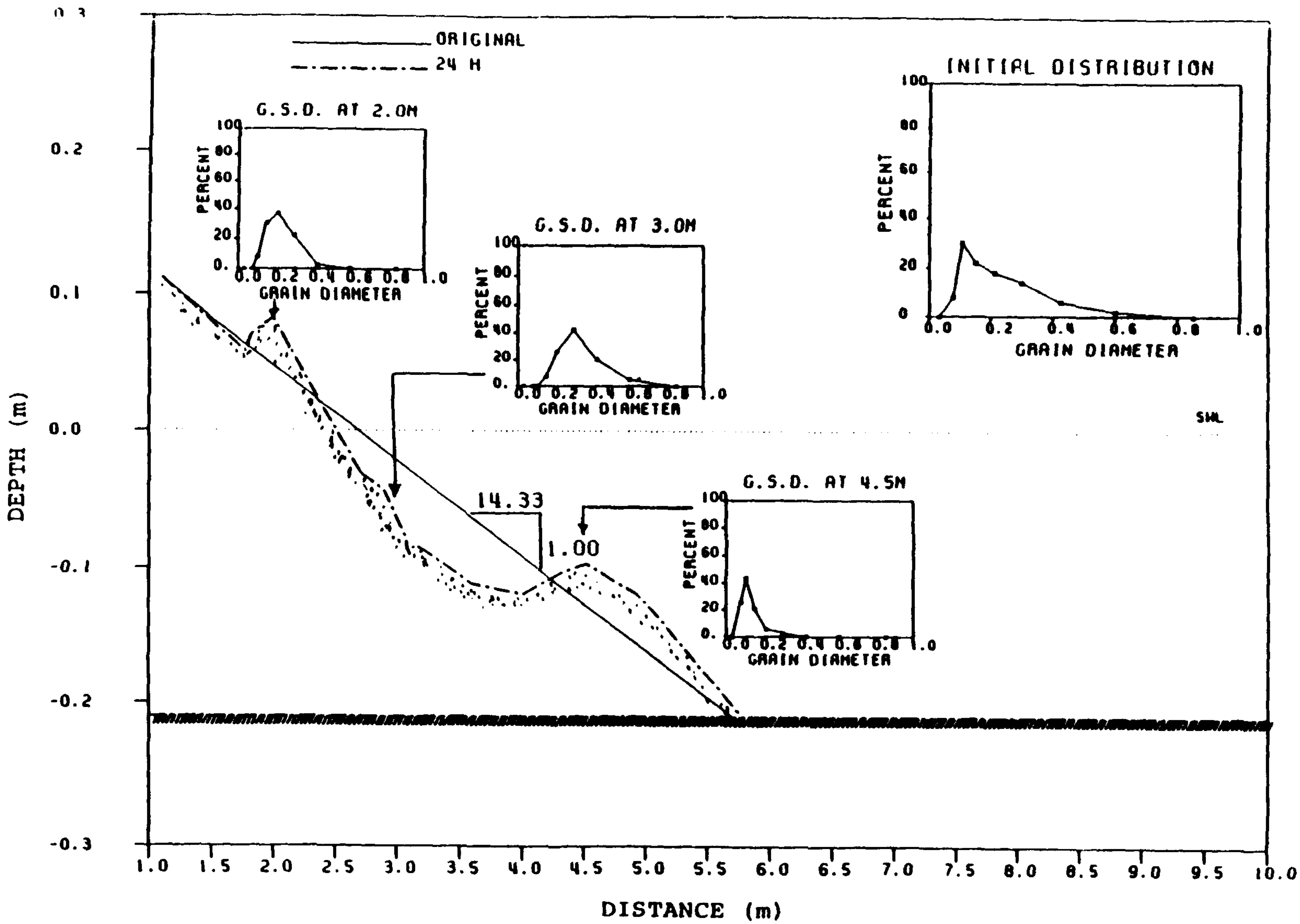


Figure 36. Experiment 6, initial and final profiles and initial and final grain size distributions

Table 2
Results of the Tracer Analysis

Time	Type of Observation	Orange	Green	Red	Yellow	Magenta
1 HR	Visual					
	At 1.5 m	Dyed zone				
	At 1.6-1.7 m		Dyed zone			
	At 1.7-2.0 m			Spots		
	Sample tubes					
	At 1.5 m	On the top				
	At 1.7 m			Few grains		
	Accumulation tube					
	At 1.9 m	Many	Slightly dyed	Many		
	At 2.5 m	Some	Some	Some		
	At 3.0 m					
	At 3.5 m			Some		
	At 4.0 m	Slightly dyed	Slightly dyed	Many		
	At 5.0 m				Few	Many
6 HR	Visual			Layers by the glass		
	Accumulation tube					
	At 2.0 m			Few		
	At 2.5 m			Few		
	At 2.9 m			Some		
	At 3.1 m			Few		Few
	At 3.2 m			Few		
24 HR	Sample tubes					
	At 1.95 m	Layer at 4 cm below top		Few		
	At 2.30 m	Few at 1-5 cm from top				

(Continued)

Table 2 (Concluded)						
Time	Type of Observation	Orange	Green	Red	Yellow	Magenta
	At 2.50 m	Layer at 1.5 cm below top		Few		
	At 2.80 m		Few top	Few top		
	At 3.00 m			3 Grains		
	At 3.50 m		Few		Few	
	At 3.80 m					
	At 4.20 m					Few
	At 4.60 m					Few
	At 5.10 m					Layer at 3.5 cm below top

c. Problems similar to those with the blue tracer could have occurred with the orange; however, as the placement of the orange was closer to the shoreline, the area over which it was spread was smaller, and the concentration higher. This allowed these particles to be identified upon the conclusion of the experiment.

d. The tracers that were initially located at the shoreline were transported and sorted on the beach face during the first hour of wave action. They remained there until the end of the experiment. At that time, they were located 4 cm below the surface corresponding to the profile during the first hour.

e. The orange tracer was found at the top of the berm while the red, not so evenly spread, was found at the low part of the beach face. Recall that the orange tracer was the finest and that the red was the coarsest.

f. At the bottom of the profile, just before the offshore bar, tracers of different colors were found, indicating that transport has occurred in both directions--seaward and landward--of this point.

g. No other evidence was found to indicate major patterns in sediment transport.

Other results concerning all experiments are presented below and will be used later to develop additional conclusions.

Figures 20f and 37 present the remaining grain size information for Experiment 6.

By comparison, the profiles equilibrated after 12 hr of wave action and then, only approximately. However, the main features of the final profile were established very fast; that is, the beach face slope, the sandbar, and the berm were formed in the first 6 hr and after that, they only migrated or modified their volumes slightly. Layers of sand were added or removed from the existing features, but the slopes that dominate the main characteristics of the beach remained almost constant. The tracers (Experiment 6) only confirmed the way in which the main beach features developed. At the very beginning of the experiment, the waves carried the finest tracers up the beach face. Furthermore, the coarsest tracer type was spread over a larger area. After some hours, a berm had been formed and the beach face became steeper. The sediments were no longer able to be transported up the beach as far as before, so the orange tracers, which were transported to the farthest onshore location, remained uncovered until the end of the experiment. On the other hand, the other tracers were covered by successive layers being found after 24 hr at some depths below the surface.

Comparison of Laboratory Data with Predictions

In this section, the six "final" (i.e. 24-hr) profiles obtained in the laboratory experiments were compared with computed profiles based on the cross-shore varying mean grain sizes as documented in the laboratory. It is stressed that the comparisons to be presented are "blindfolded" in the sense that they do not incorporate any calibrations or adjustments to improve the fit. Rather, the computed profiles are based on the empirical relationship between the sediment scale parameter A, and the sediment size D, presented in Figure 2. Comparisons are presented for: (a) profiles for parameterized fits to the actual A versus y distributions, and (b) profiles for the local A values.

Comparison Based on Parameterized Fit to the Actual *A* Values

The method will first be described, then the results of the comparison presented.

The procedure considers cross-shore variations of A of the forms

$$A(y) = A_0 \, e^{-ky} \tag{9}$$

$$A(y) = A_0 + my \tag{10}$$

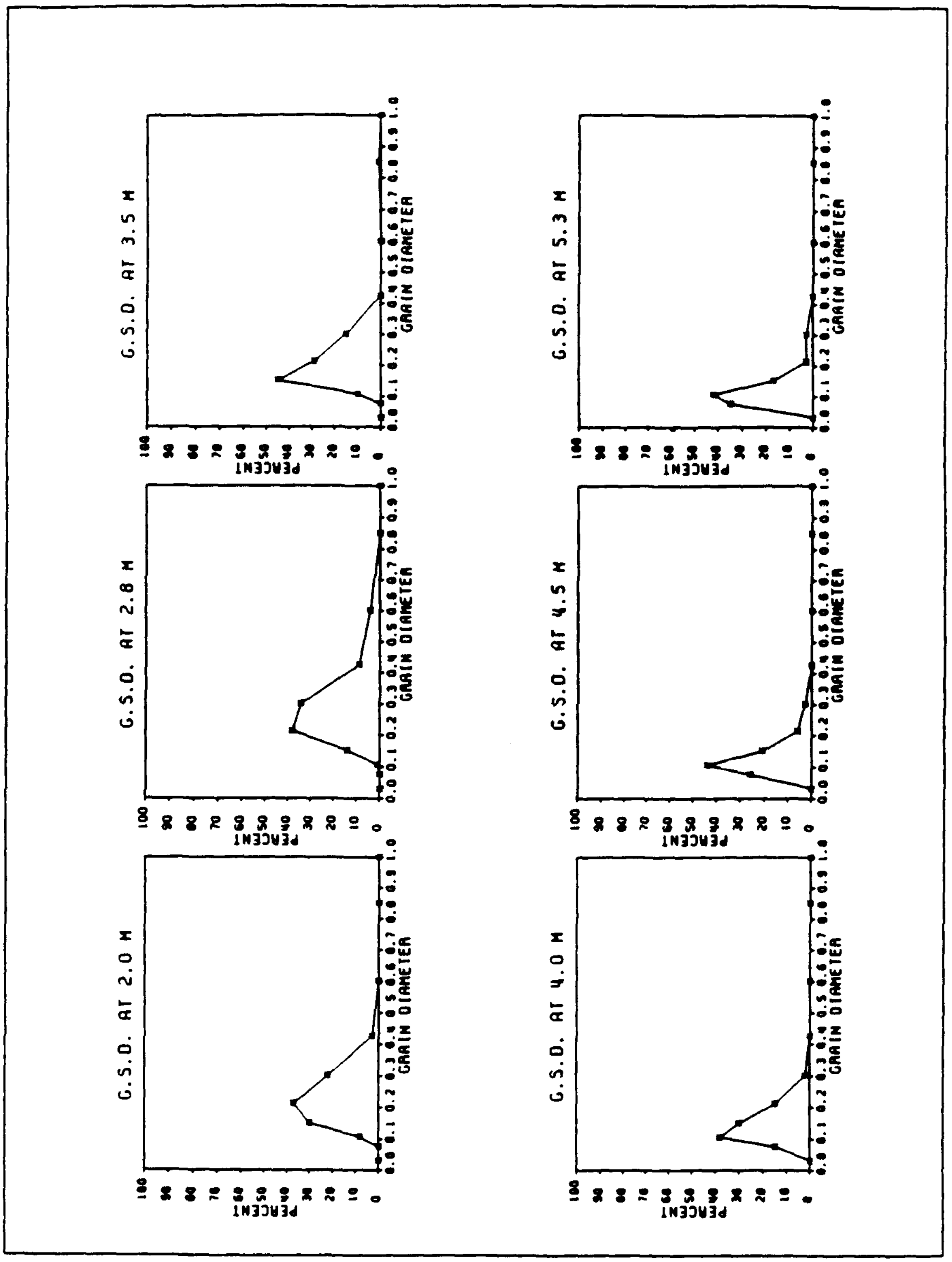


Figure 37. Experiment 6, grain size distributions at six locations across the profile after 24 hr

in which y is the distance offshore from the still-water line, A_o is the (idealized) sediment scale parameter evaluated at the shoreline, and k and m are empirical coefficients describing the best-fit variation in the cross-shore direction.

The general expression for equilibrium beach profiles is

$$\frac{dh^{3/2}}{dy} = A^{3/2} \tag{11}$$

in which A is the local value of the sediment scale parameter. Substituting Equation 9 into Equation 11 and integrating

$$h(y) = A_0 \left\{ \left(\frac{2}{3k} \right) (1 - e^{-3/2\, ky}) \right\}^{2/3} \tag{12}$$

and considering the variation given by Equation 10,

$$h(y) = \left\{ \frac{2}{5m} \left[(A_0 + my)^{2.5} - A_0^{2.5} \right] \right\}^{2/3} \tag{13}$$

The method described above was applied as follows. The distribution of the A parameter across the profile was determined for each of the final sediment samples based on the mean diameter and transforming to A via Figure 2. Next, the best least-squares representations of these data by Equations 9 and 10 were established. Finally, the calculated profiles were based on Equations 12 and 13. Figures 38-43 present a comparison of the calculated and measured profiles for the six experiments. These results are discussed briefly below.

For Experiments 1, 5, and 6, the predicted profile is somewhat steeper than the measured, whereas for Experiments 2 and 3, the measured is steeper than the predicted and for Experiment 4, there is reasonable agreement.

Comparisons Based on Localized *A* Values

In this comparison, which was only applied to Experiments 1, 2, and 3, the calculated profile was based on the local A(y) value as given by

$$h(y+dy) = \left[h^{3/2}(y) + A^{3/2}(y)dy \right]^{2/3} \tag{14}$$

which follows directly from the differential form (Equation 11). Results of applying Equation 14 to calculate profiles are presented in Figures 44 and 45.

Results are somewhat similar to those presented and described for the method using parameterized fits to the actual A versus y distributions and will not be discussed further.

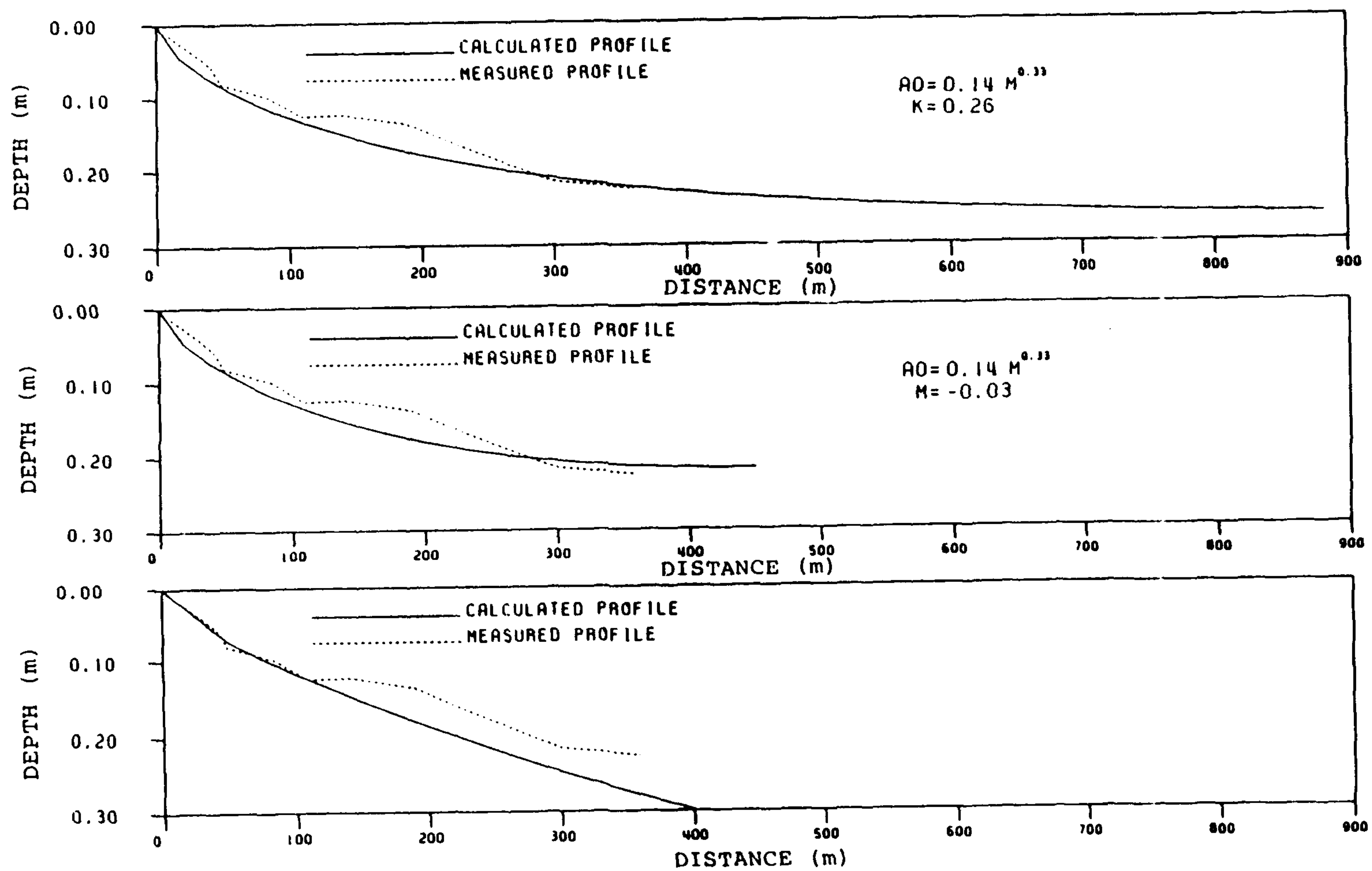


Figure 38. Experiment 1, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

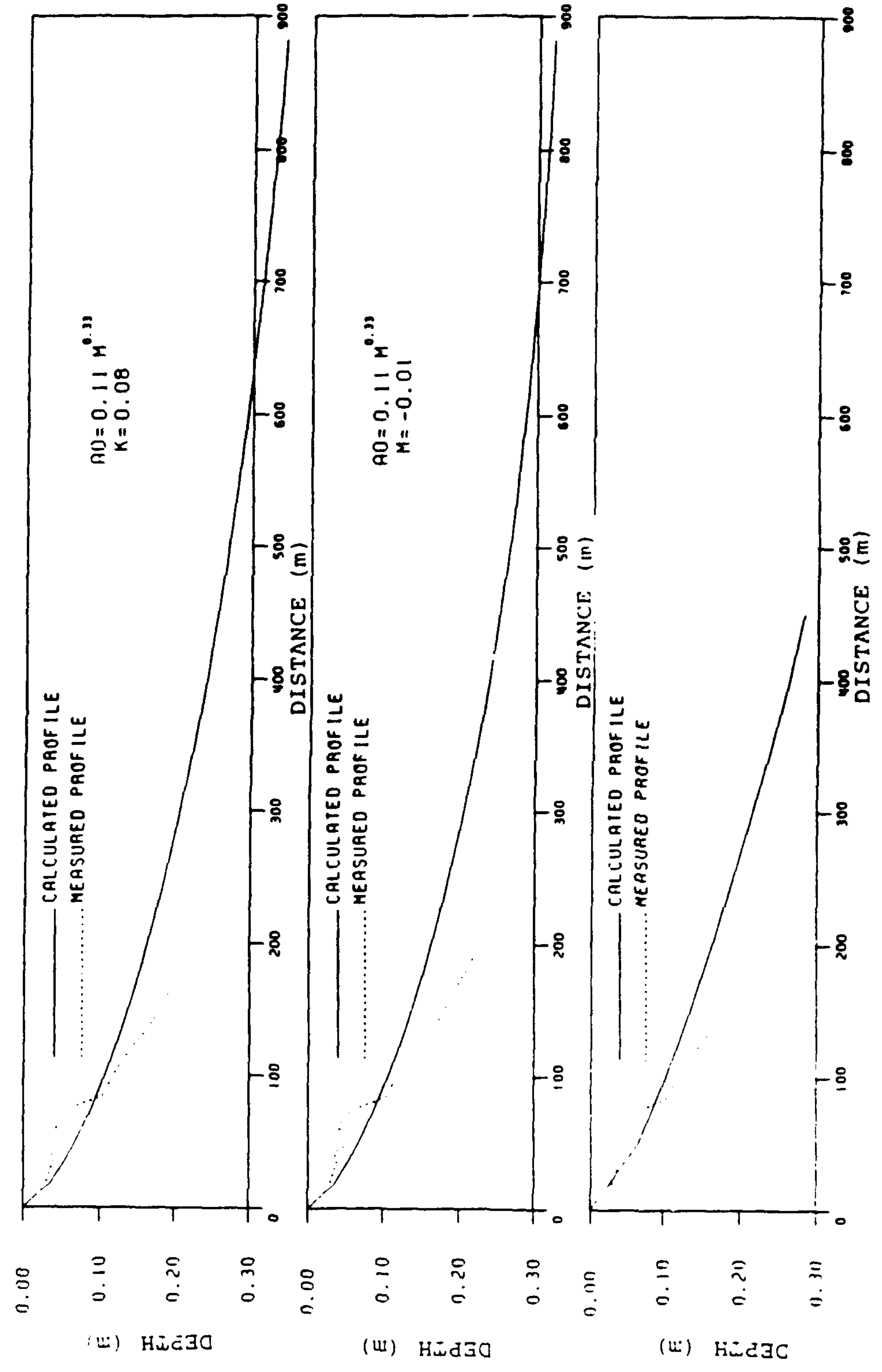


Figure 39. Experiment 2, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

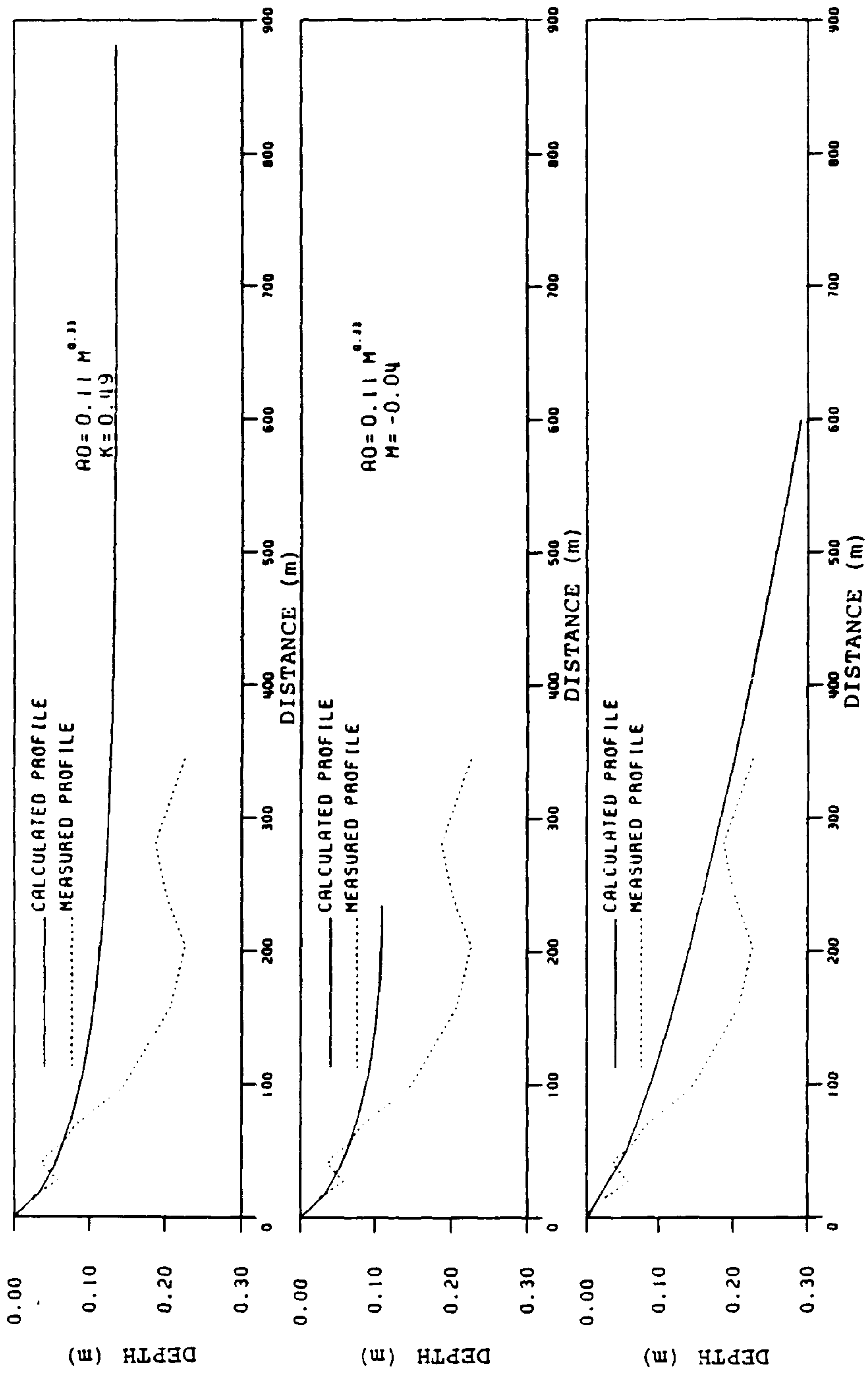


Figure 40. Experiment 3, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

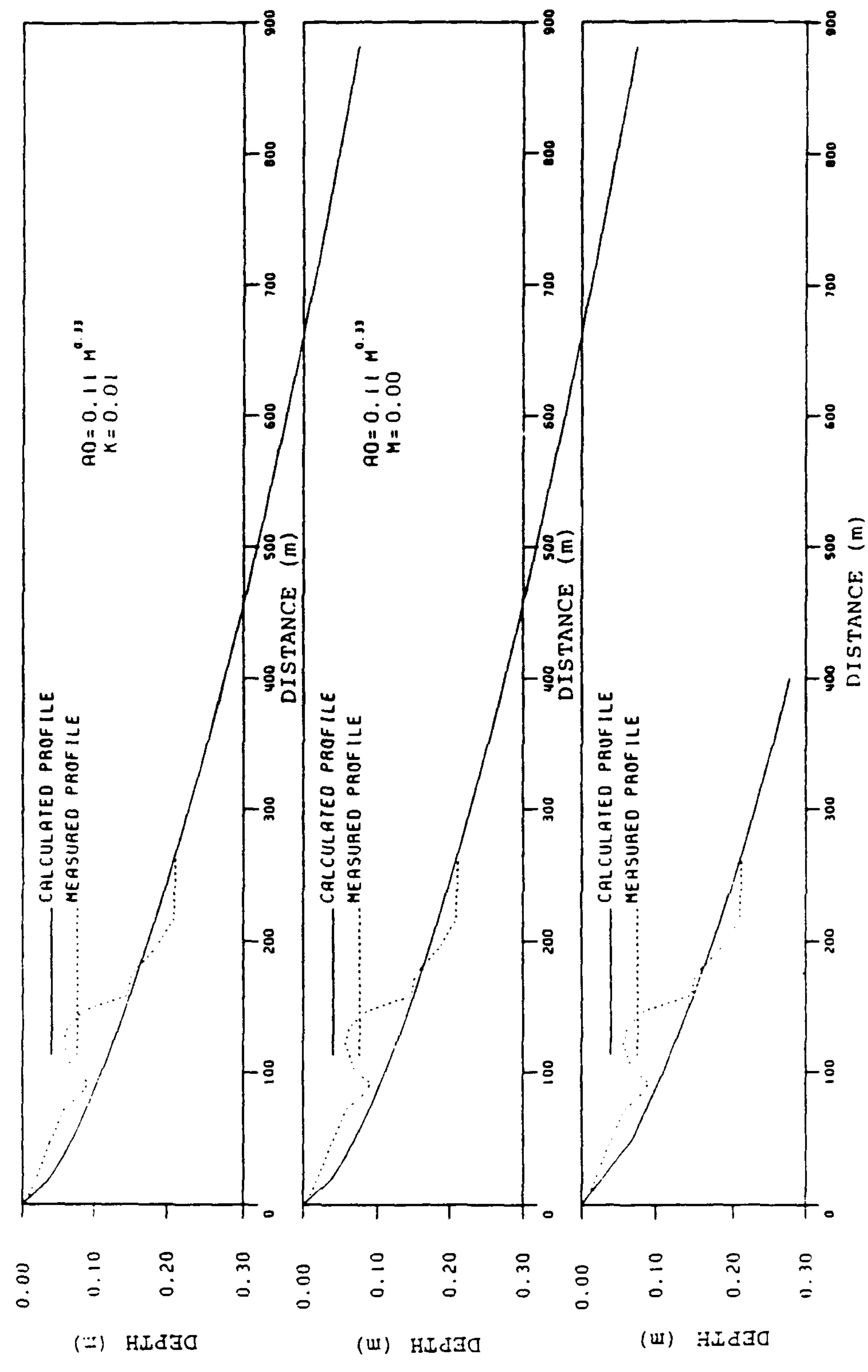


Figure 41. Experiment 4, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

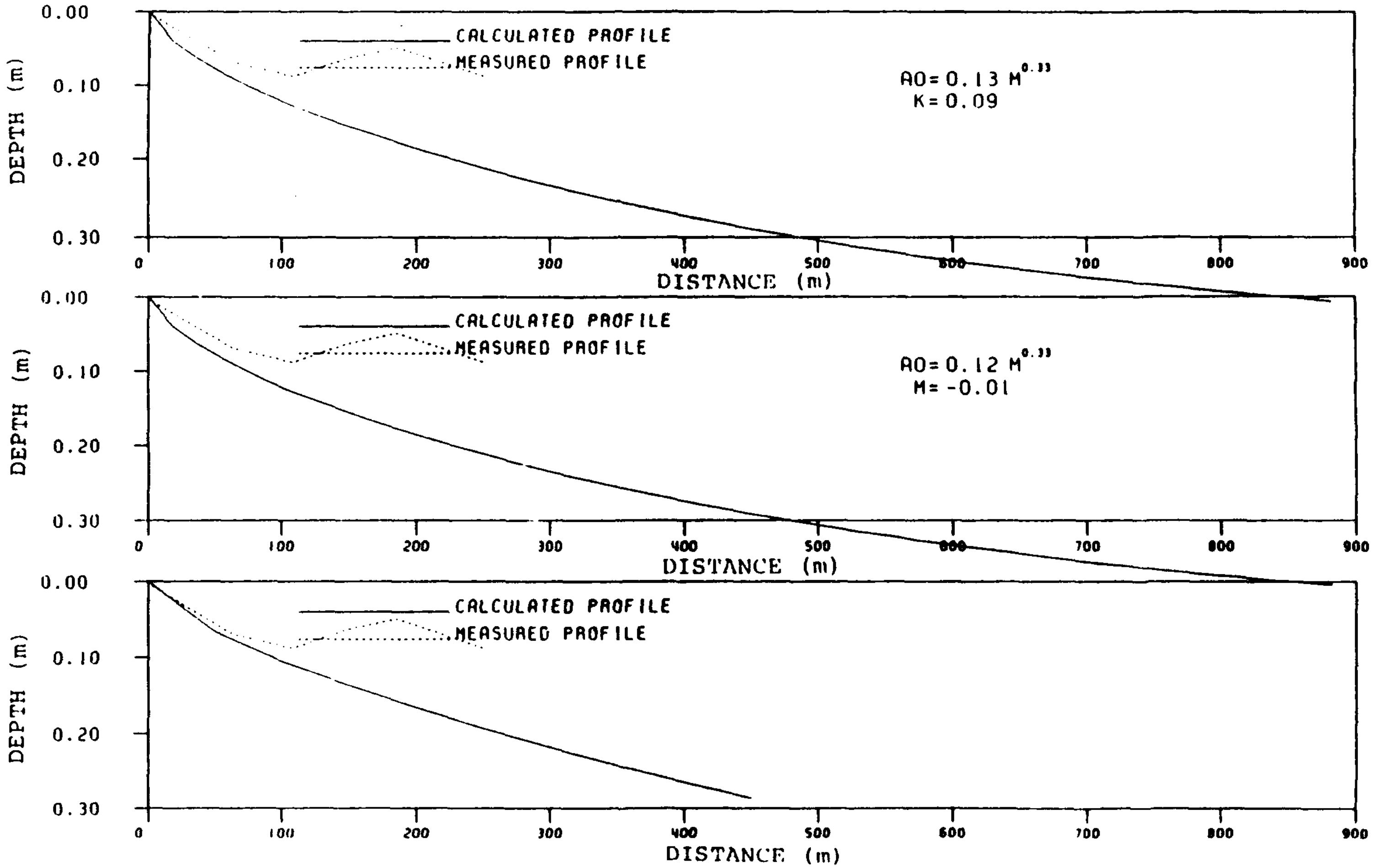


Figure 42. Experiment 5, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

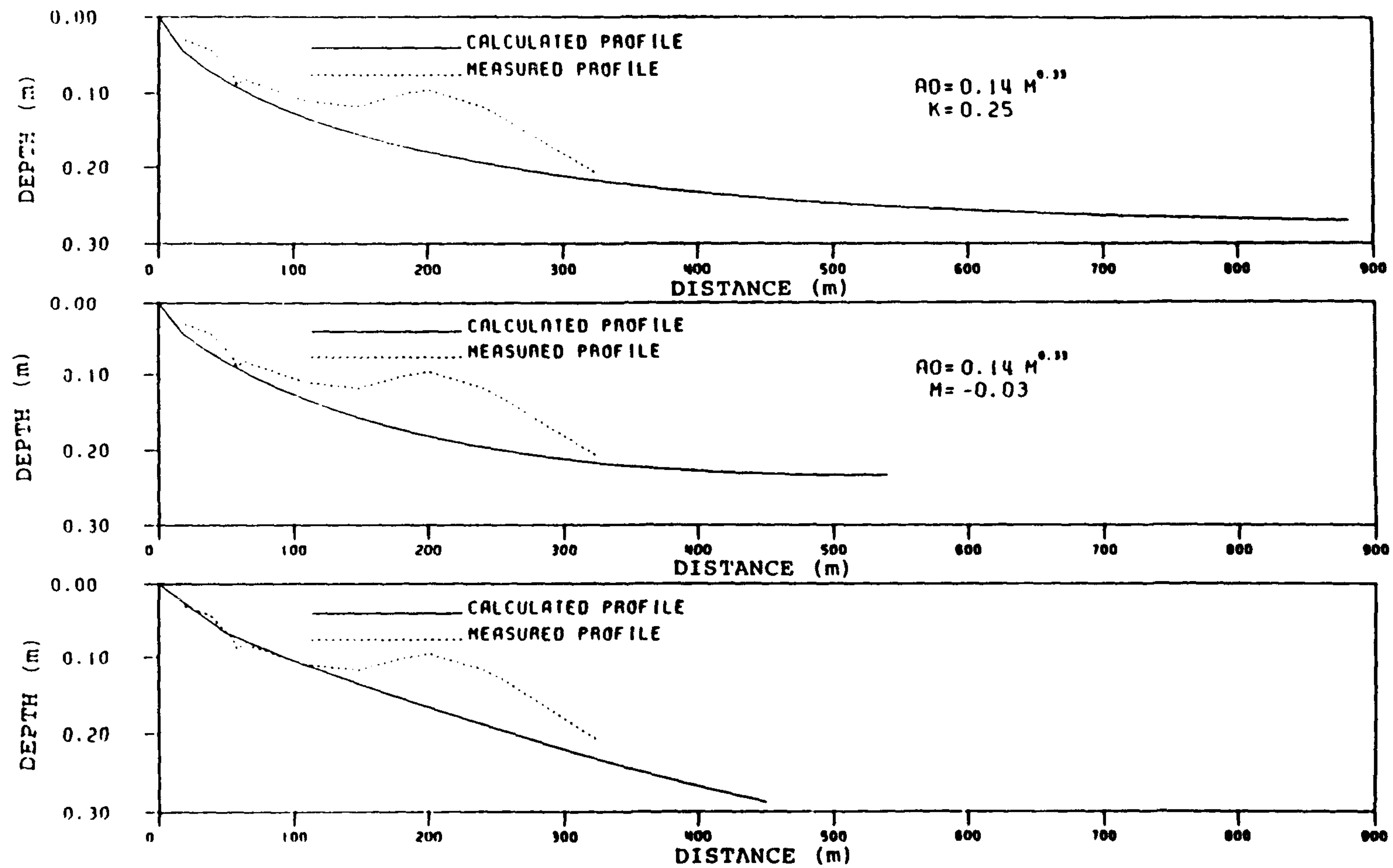


Figure 43. Experiment 6, equilibrium beach profiles. Top: exponential variation of A; middle: linear variation of A; bottom: Average A

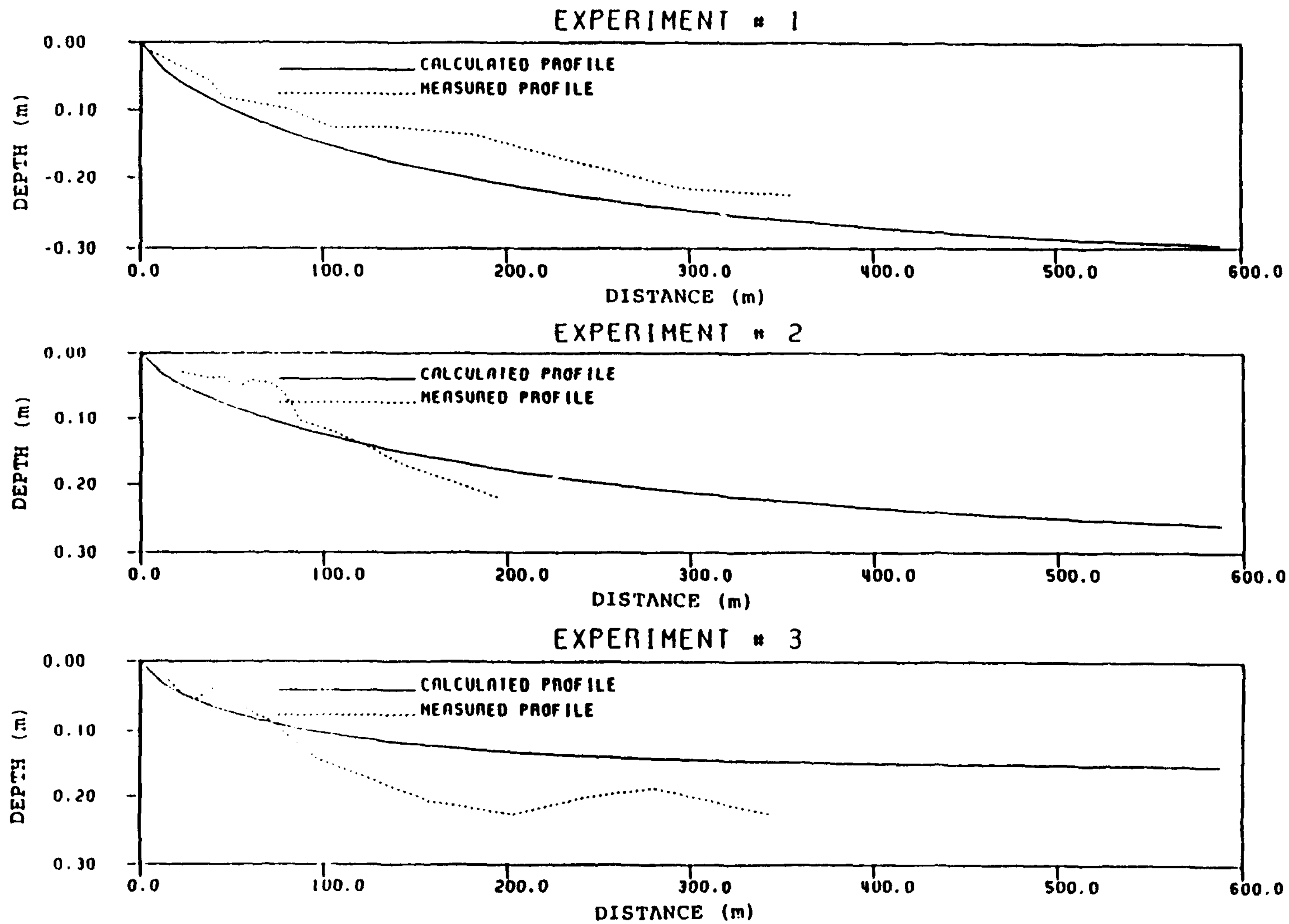


Figure 44. Experiments 1, 2, and 3, comparison of predicted and measured profiles for exponential fit to *A* values as determined from measured mean sediment sizes

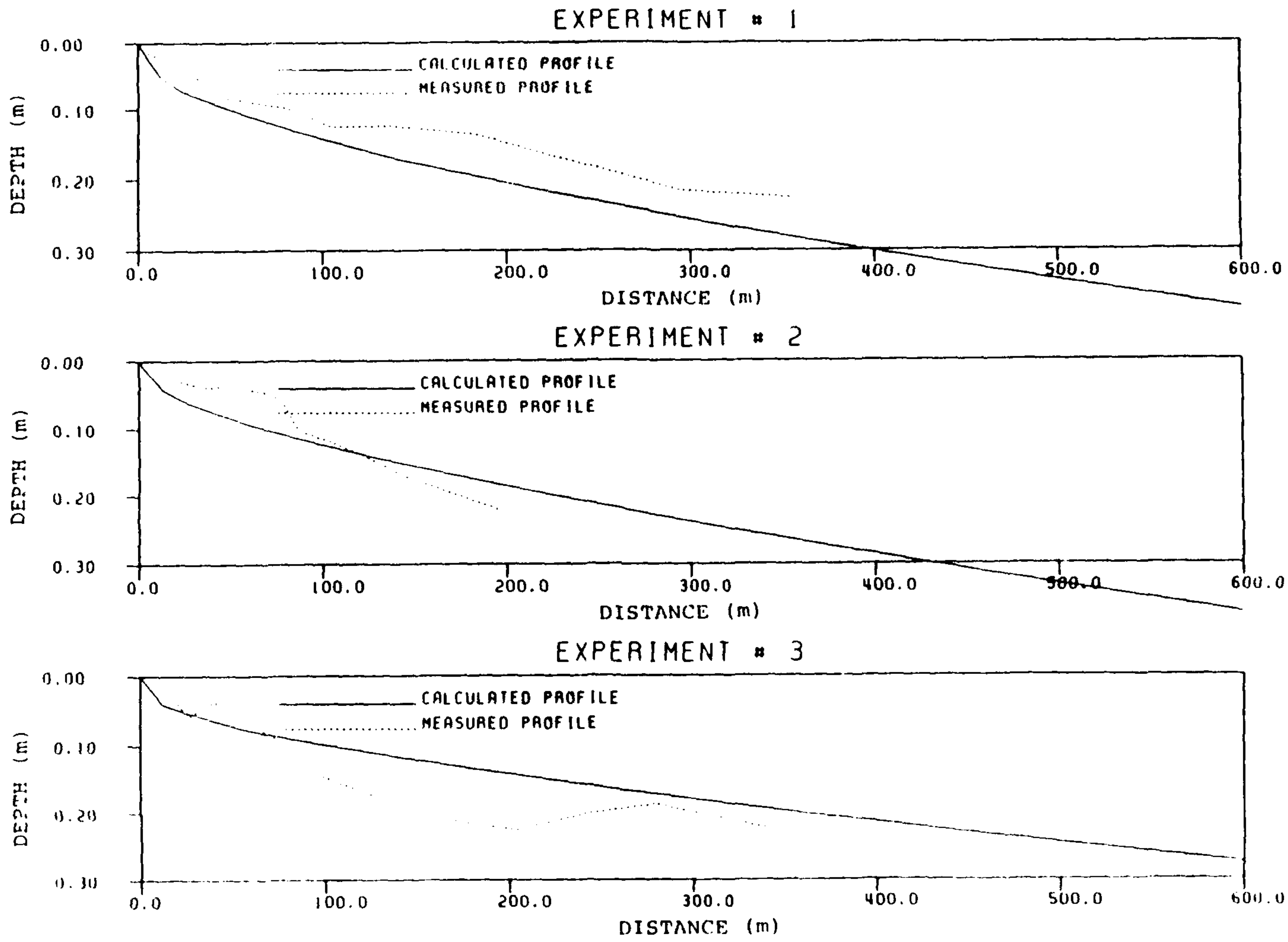


Figure 45. Experiments 1, 2, and 3, comparison of predicted and measured profiles for local *A* values as determined from measured mean sediment sizes

5 Field Data

Two sets of field data were selected for comparison and evaluation using the general methodology presented. Field data sites were Delray Beach, Florida, and Jupiter Island, Florida; both sites have experienced multiple renourishment events. As with most field results, the data sets are not as complete as desired. However, even though incomplete and complex, field data are valuable, as they contain no scale effects. The following sections present field data from these two sites and utilize the available data to evaluate the general methodology presented here.

Delray Beach, Florida

As shown in Figure 46, Delray Beach was nourished in 1973, 1978, and 1984. Figure 47 presents the northerly and southerly limits of the 1984 nourishment. The mean diameter of this nourishment material is reported to be 0.16 mm, whereas that of the native material is 0.22 mm.

Figure 48 portrays, for Station 180.88, the 1988 distribution of mean sediment size across the beach profile and Figure 49 presents the associated *A* parameter with an exponential fit. The same information is presented in Figures 50 and 51 for Station 184.88 and in Figures 52 and 53 for Station 187.88. Additional profile and sediment size information for Delray Beach is contained in Appendix C. Two types of profile comparisons were carried out as described below.

The first type of profile comparison is "blindfolded" in the sense that only the best-fit exponential *A* relationships (Figures 49, 51, and 53) were used with Equation 12 for the calculated profiles. These comparisons are presented in Figure 54. In general, the comparisons are considered to be quite good, with the deviations for depths greater than 4 to 5 m believed to be due to nourishment material not equilibrating to greater depths.

The second type of profile comparison is also based on exponential distributions of A values using best least squares fit to the measured profile data. These results are presented in Figure 55 and, again, the agreement is generally good.

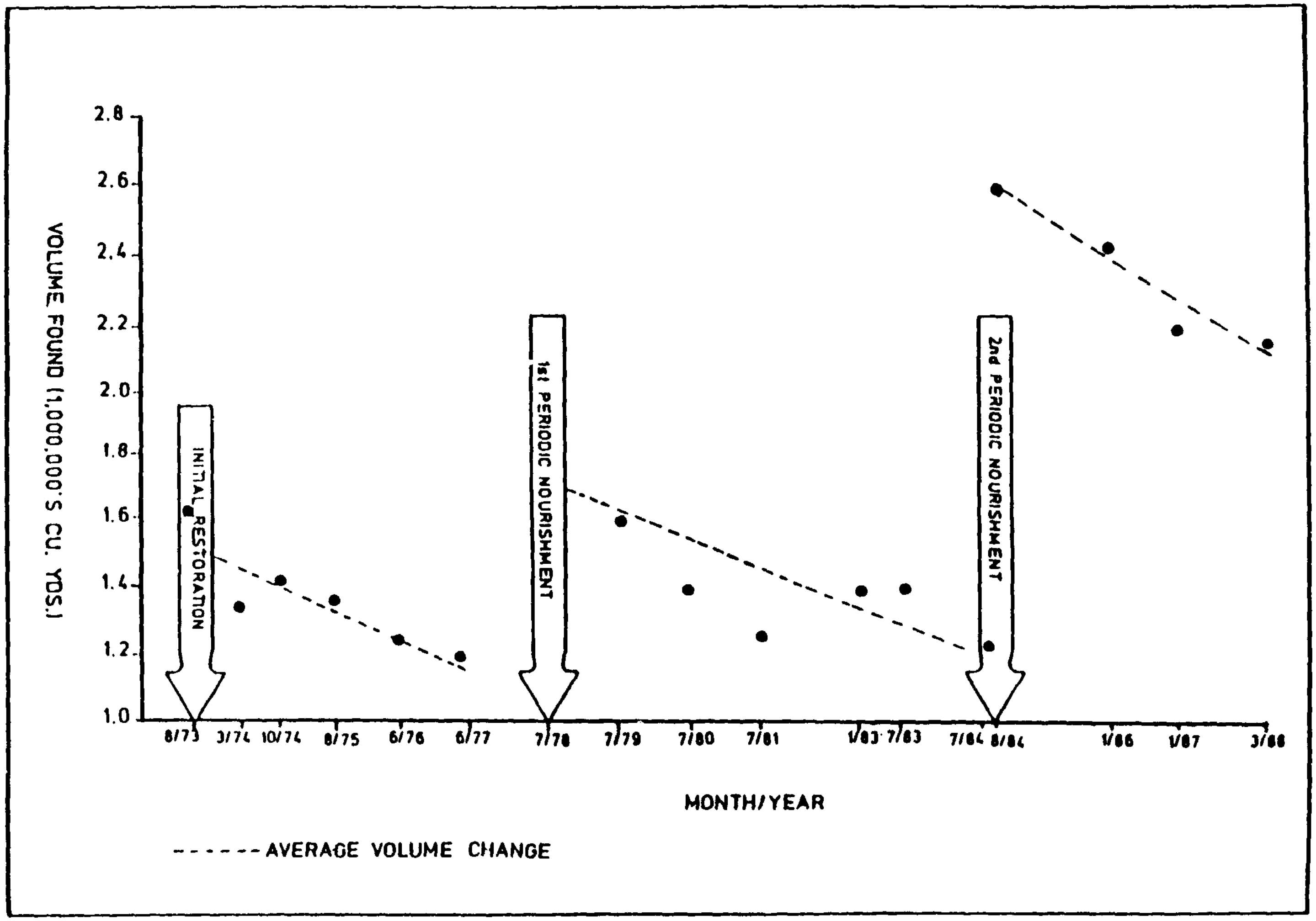


Figure 46. Nourishment events at Delray Beach, Florida, and subsequent volume changes (after Coastal Planning and Engineering, Inc. (1988))

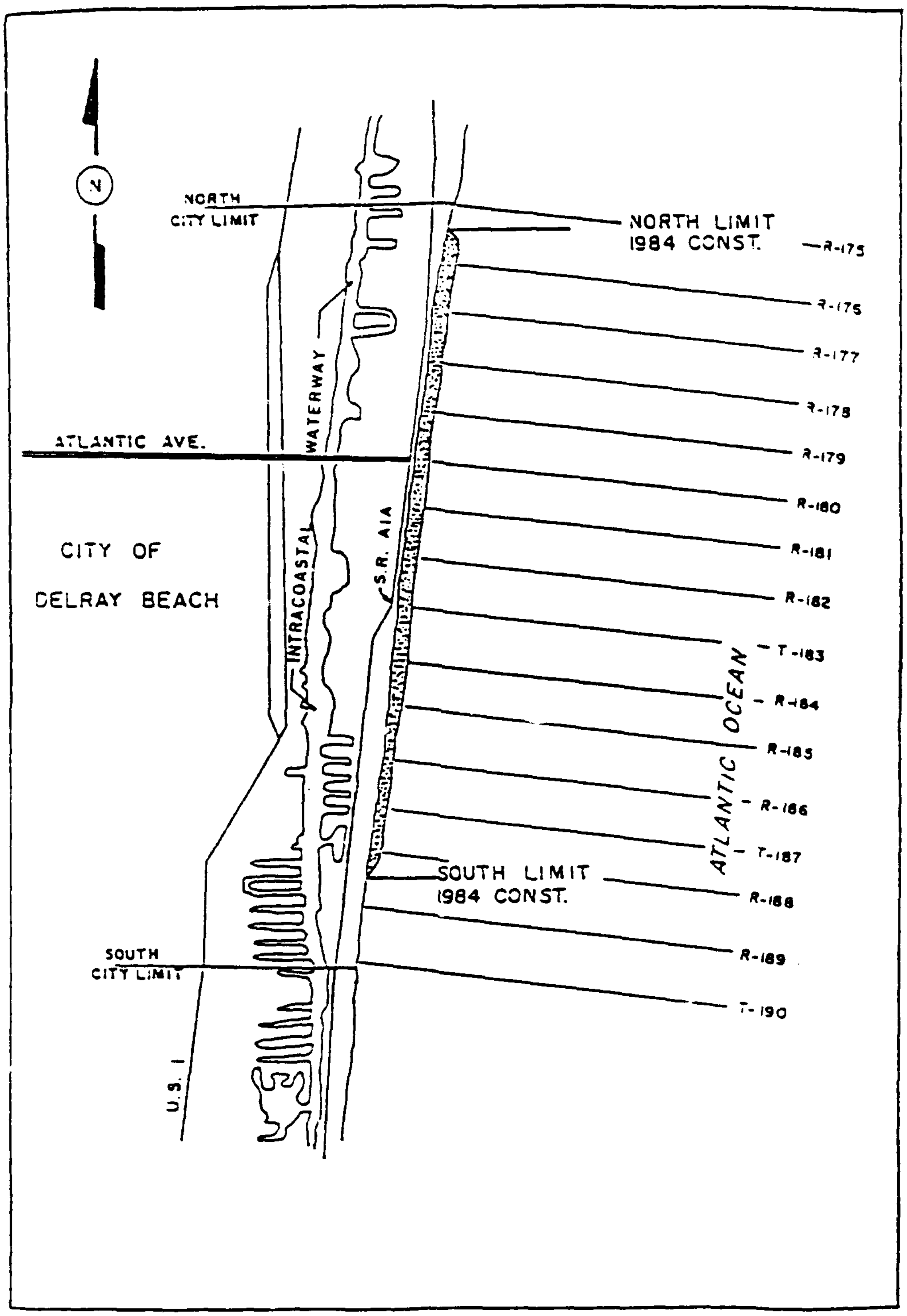


Figure 47. Location map of Delray Beach, Florida, nourishment project (after Coastal Planning and Engineering, Inc. (1988))

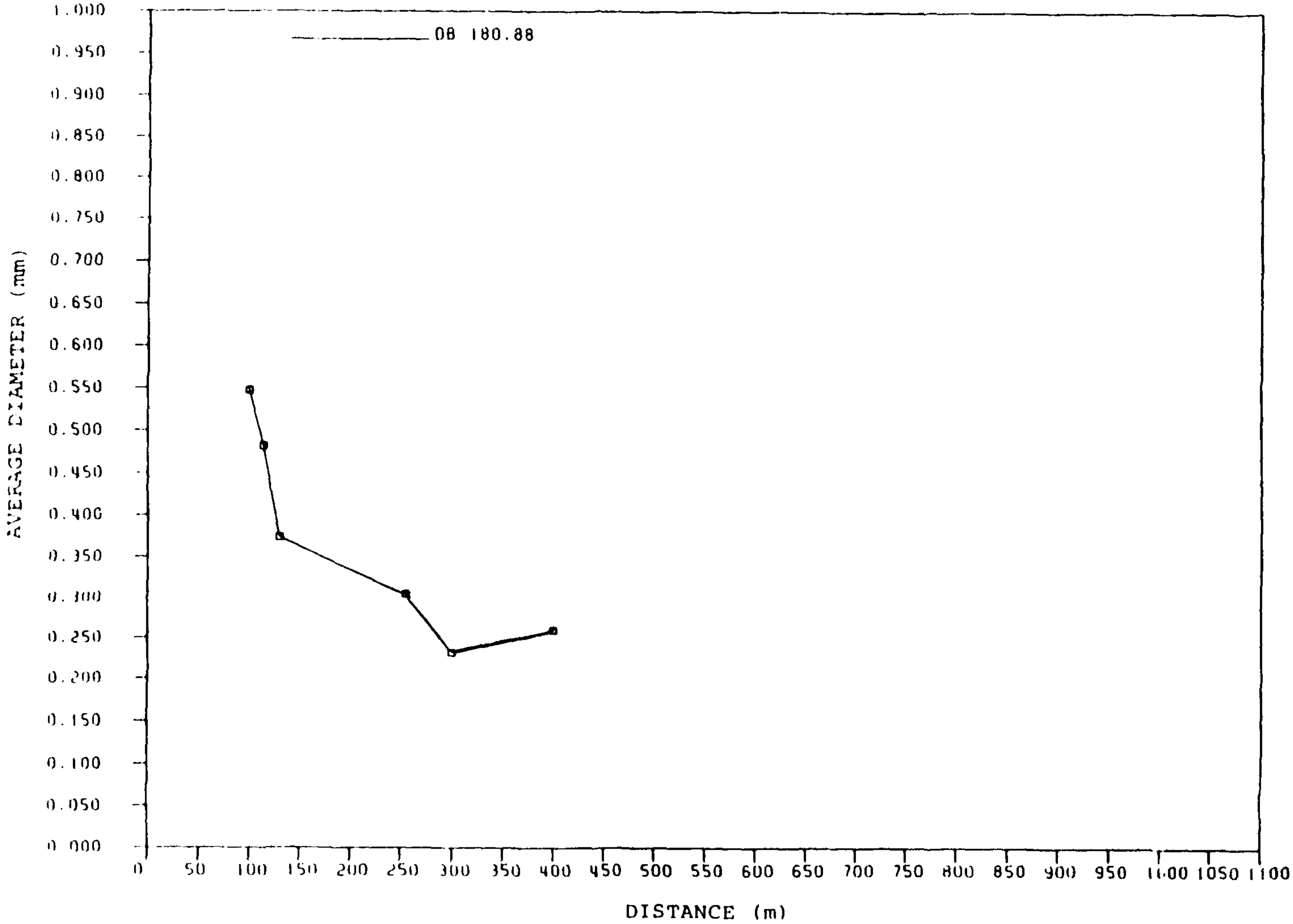


Figure 48. Average grain size variation across Profile 180.88, Delray Beach, Florida, 1988

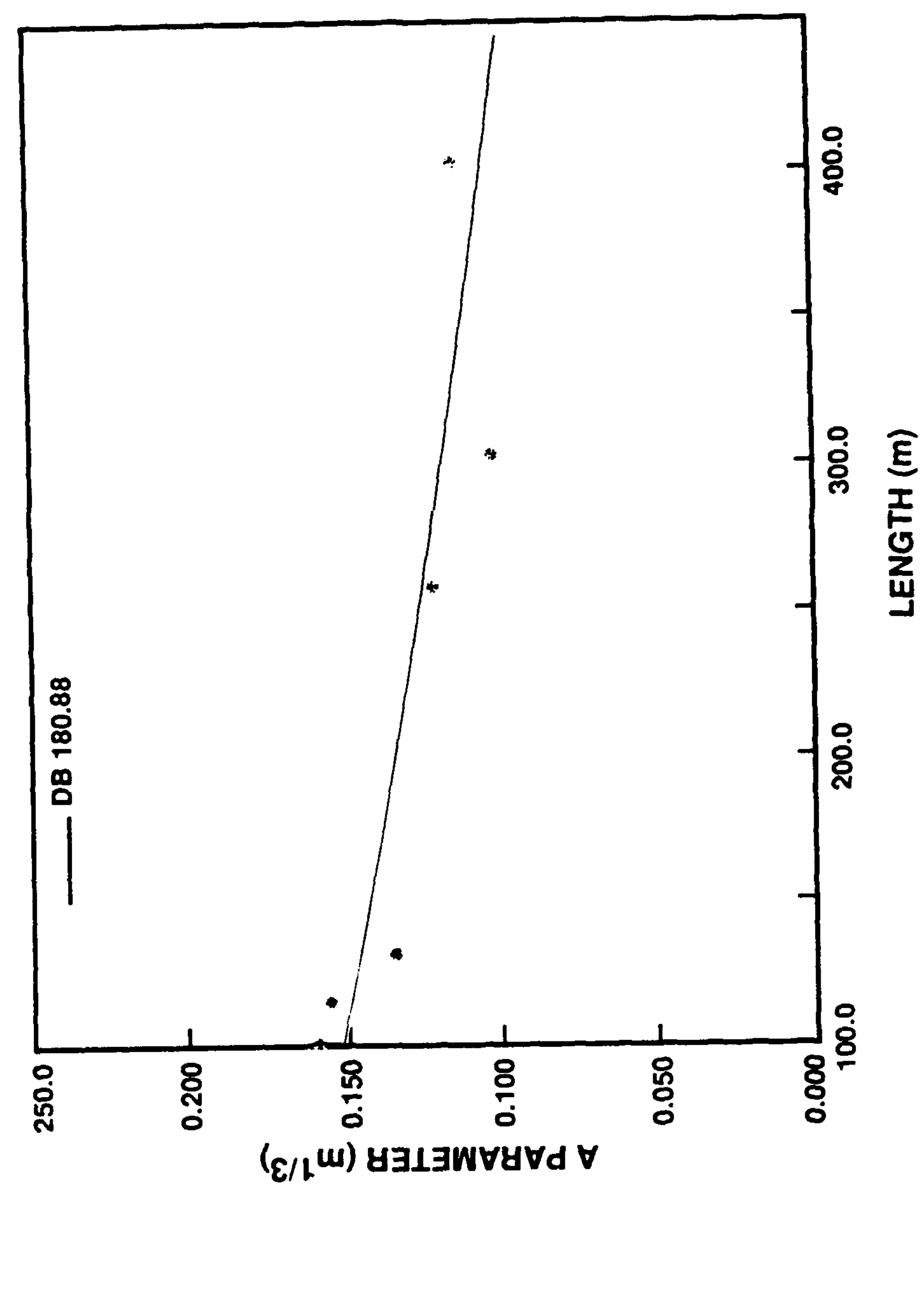


Figure 49. Exponential fit to *A* parameter distribution across Profile 180.88, Delray Beach, Florida, 1988

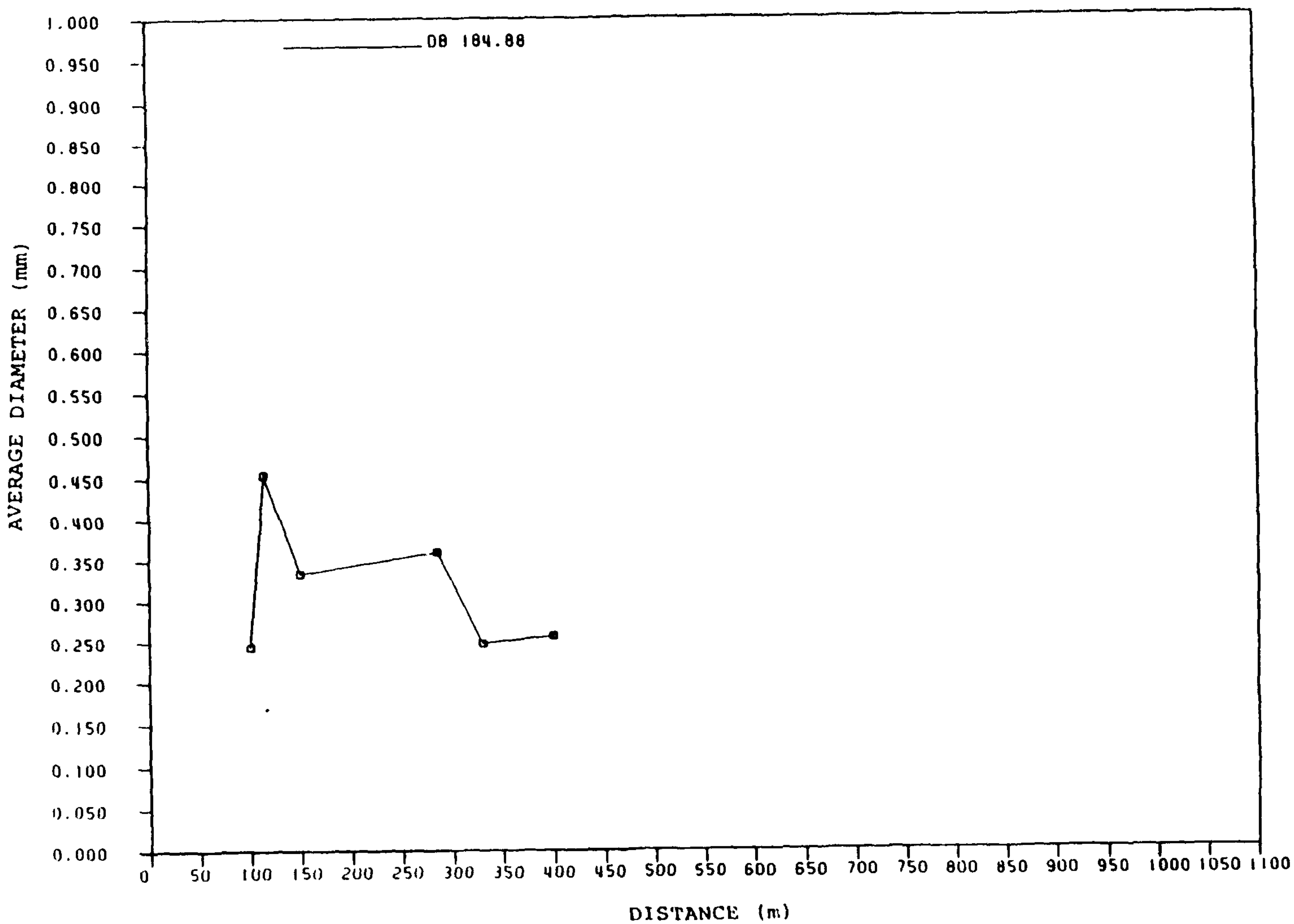


Figure 50. Average grain size variation across Profile 184.88, Delray Beach, Florida, 1988

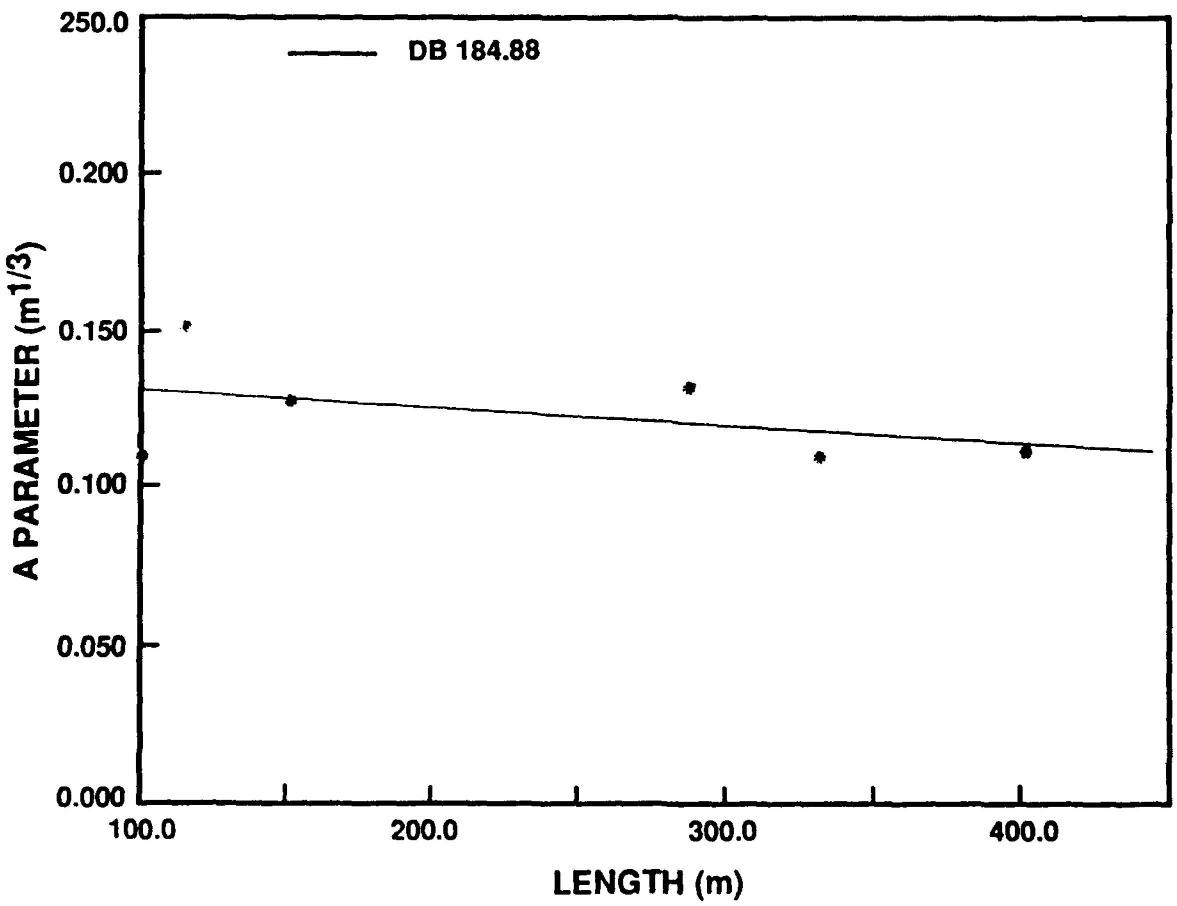


Figure 51. Exponential fit to *A* parameter distribution across Profile 184.88, Delray Beach, Florida, 1988

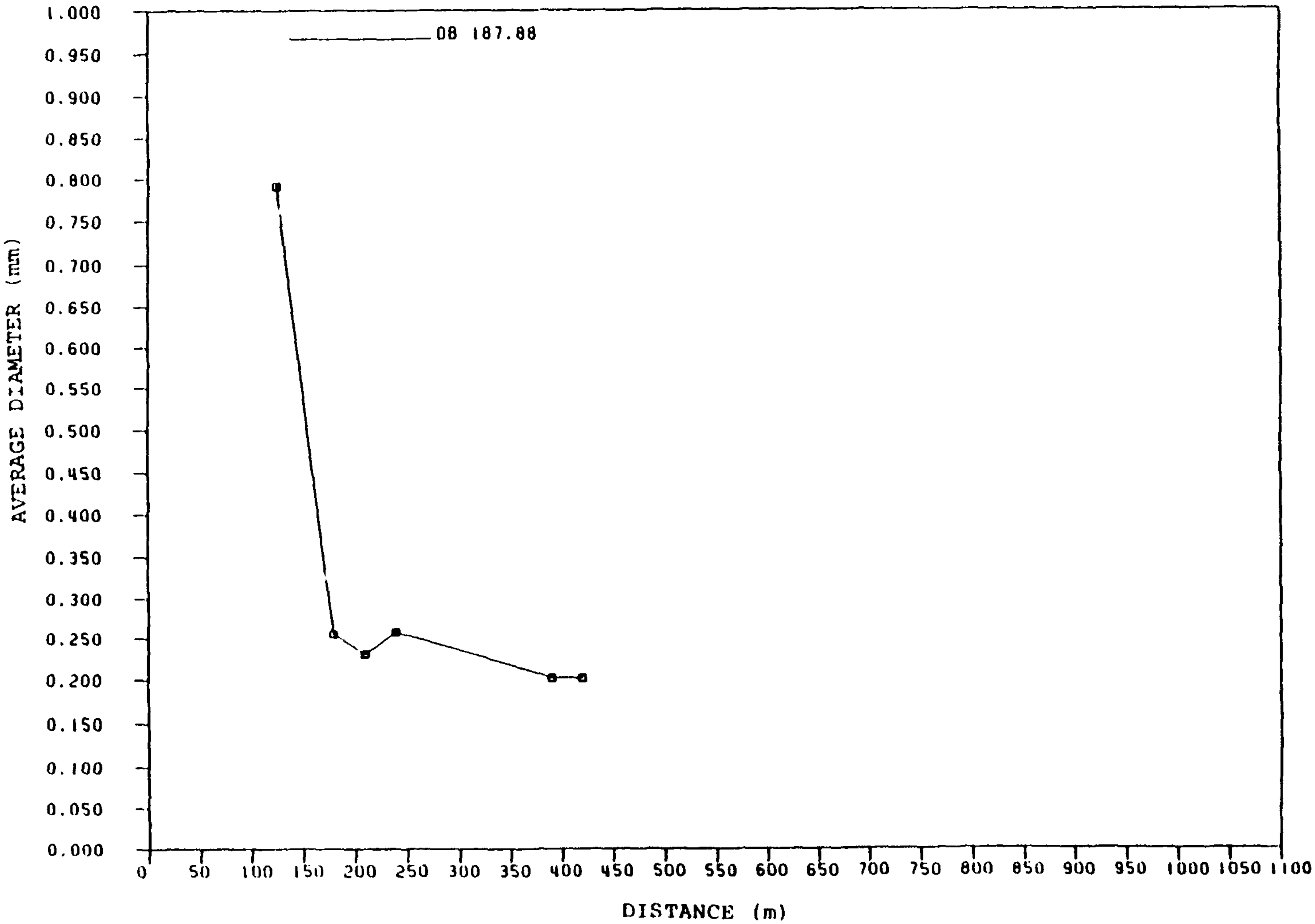


Figure 52. Average grain size variation across Profile 187.88, Delray Beach, Florida, 1988

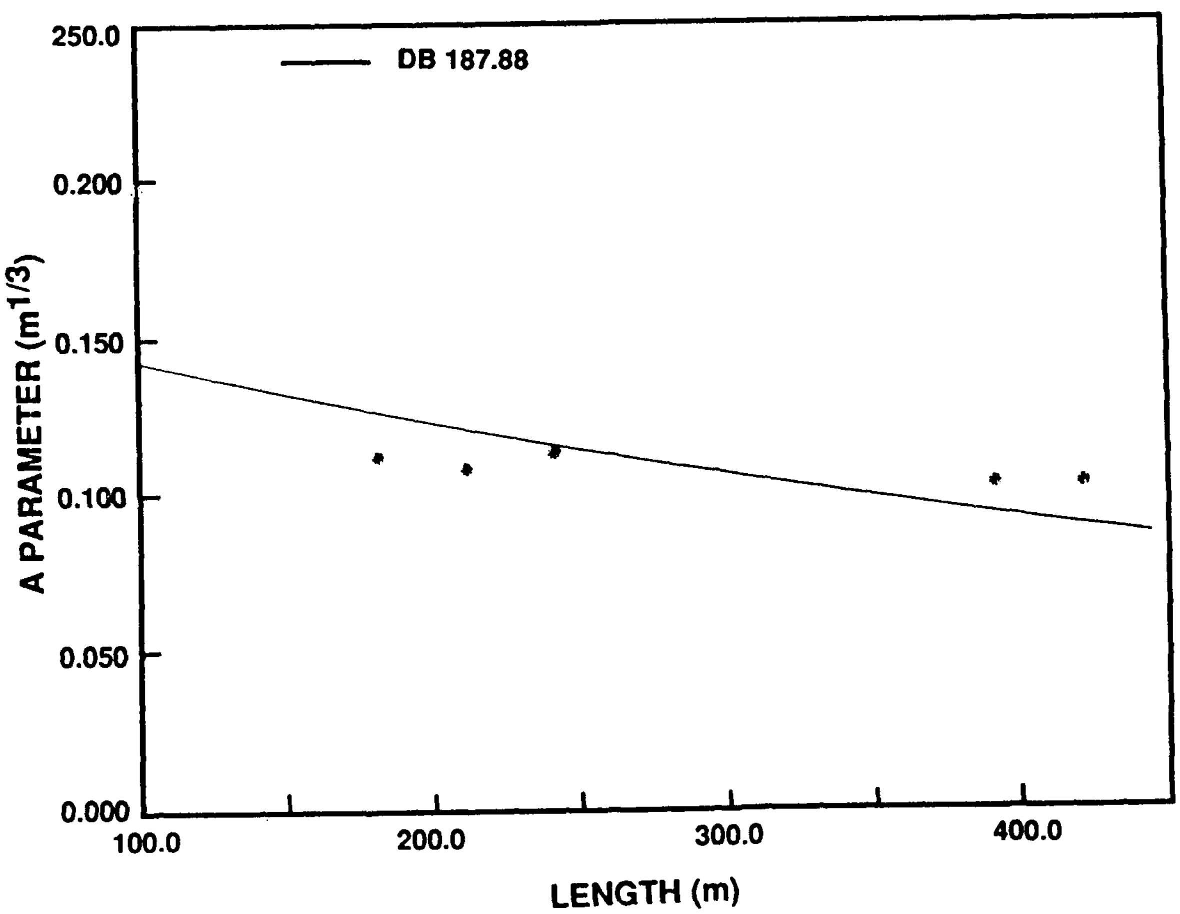


Figure 53. Exponential fit to *A* parameter distribution across Profile 187.88, Delray Beach, Florida, 1988

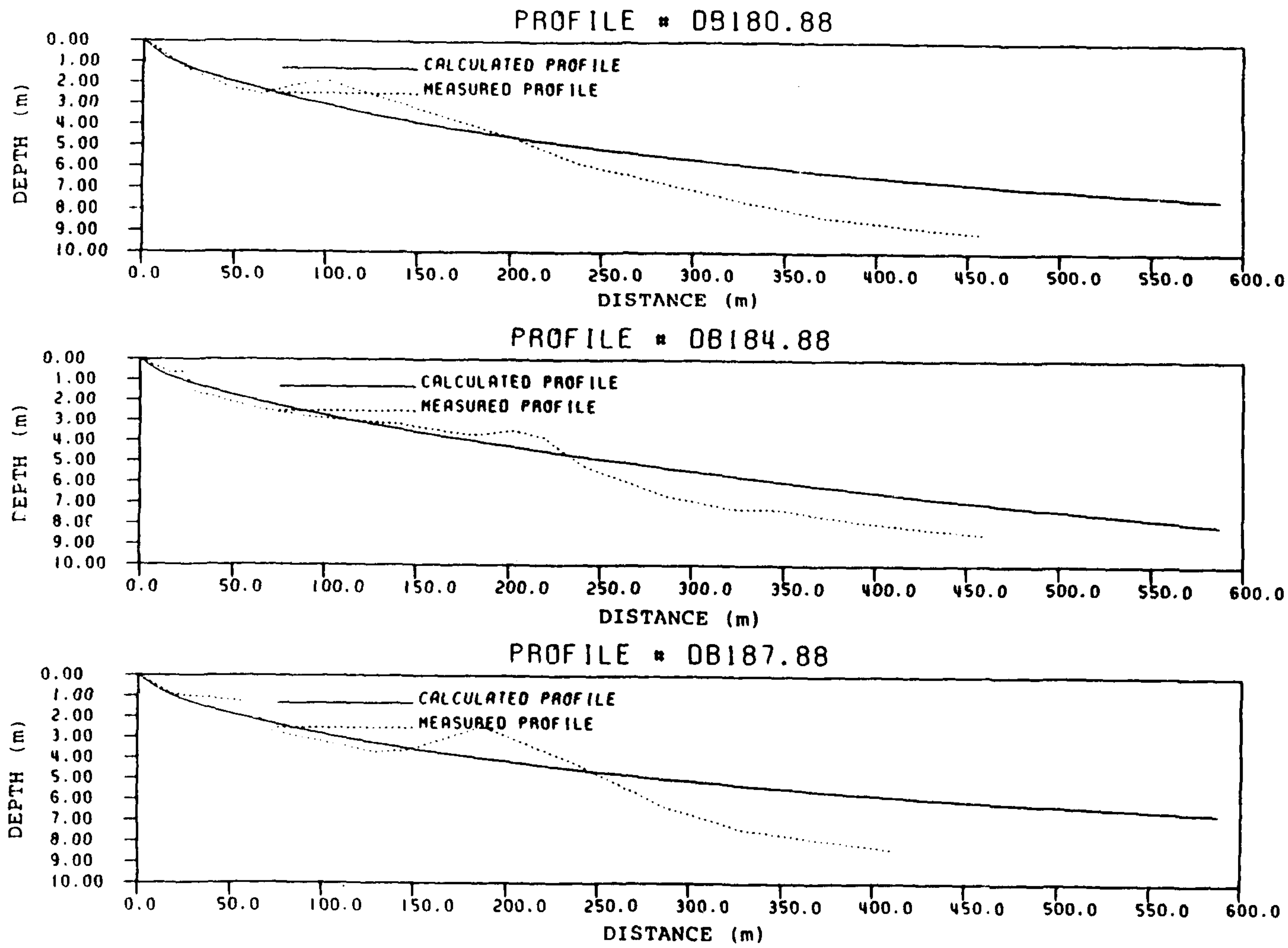


Figure 54. "Blindfolded" comparison of computed and measured profiles, Delray Beach, Florida, 1988. Computed profiles based on *A* parameter fit to measured sediment size

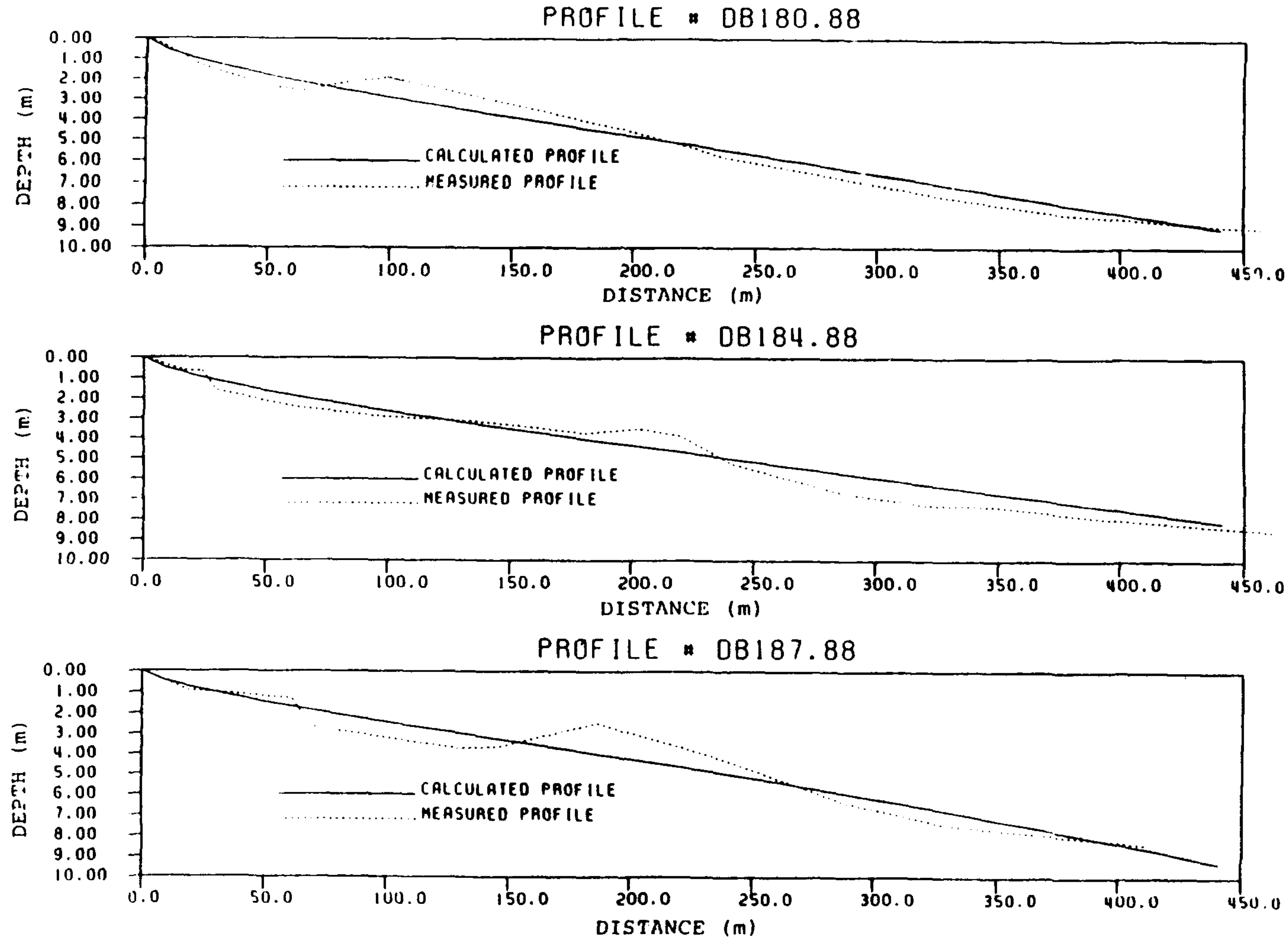


Figure 55. Comparison of computed and measured profiles, Delray Beach, Florida, 1988. Computed profiles are best fit based on exponential *A* parameter

Jupiter Island, Florida

This site has been nourished five times, with the first nourishment also occurring in 1973. The nourishment history is summarized in Table 3. Because no information is available describing the grain size distribution across the beach, comparisons are limited to information available in profiles, both pre- and post-nourishment. The nourished area and the profile designation are shown in Figure 56. The mean diameter of the nourishment material is reported to be 0.12 mm compared to the native of 0.20 mm.

Table 3
Summary of Nourishment History (Post 1973) at Jupiter Island

Year	Segment	Segment Length, m	Volume, Cubic Meters
1973	1	5,120	1,850,000
1974	1	2,800	750,000
1977	1	790	200,000
1977	2	1,100	163,000
1978	1	2,330	650,000
1983	1	1,780	454,000
1983	2	960	311,000
1987	1	950	287,000
1987	2	3,300	1,067,000
1987	3	1,080	353,000
		Total	6,085,000

Figure 57 presents best least squares fits to the 12 available 1987 measured profiles in which an exponential variation of A with offshore distance has been utilized. The general fit to the measured profiles is considered good, although at some profiles there is a nearshore rock reef (e.g., Profile J13.87, Figure 57, sheet 2) which protrudes above the sand surface.

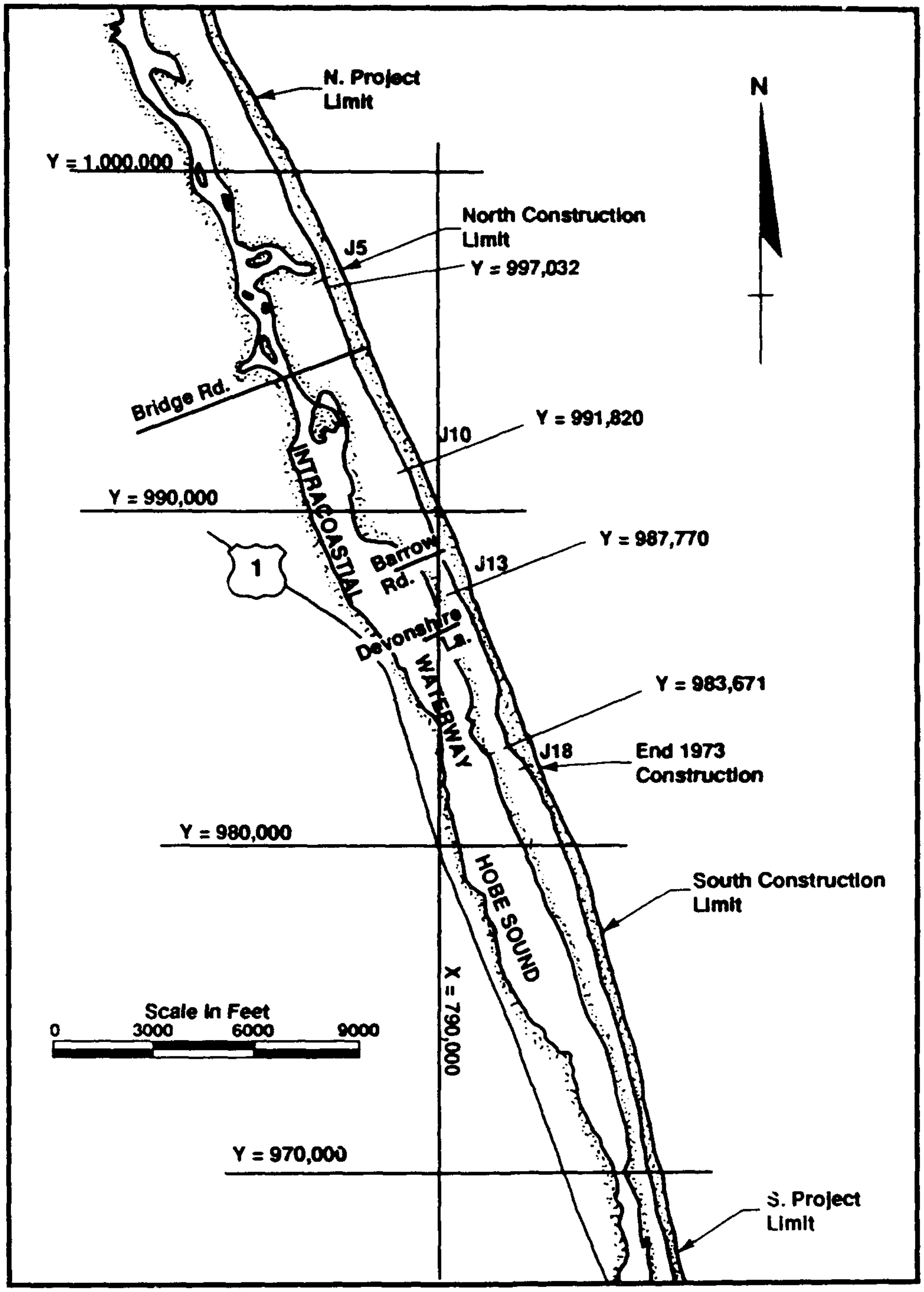


Figure 56. Project limits and profile designations, Jupiter Island project (after Strock, Arthur V., and Associates, Inc. 1981))

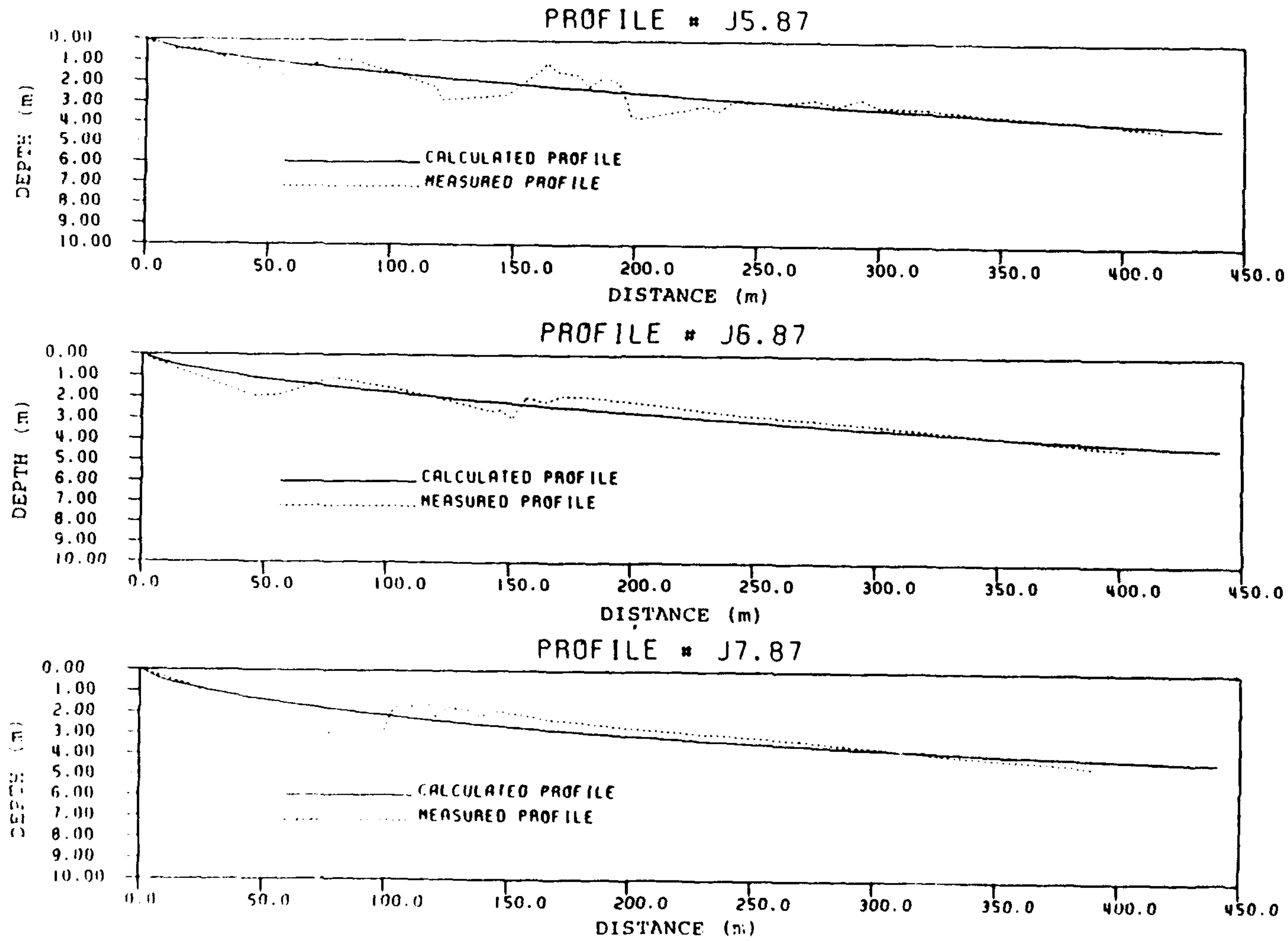


Figure 57. Comparison of computed and measured profiles, Jupiter Island, Florida, 1987. Computed profiles are best fit based on exponential *A* parameter distribution (Sheet 1 of 4)

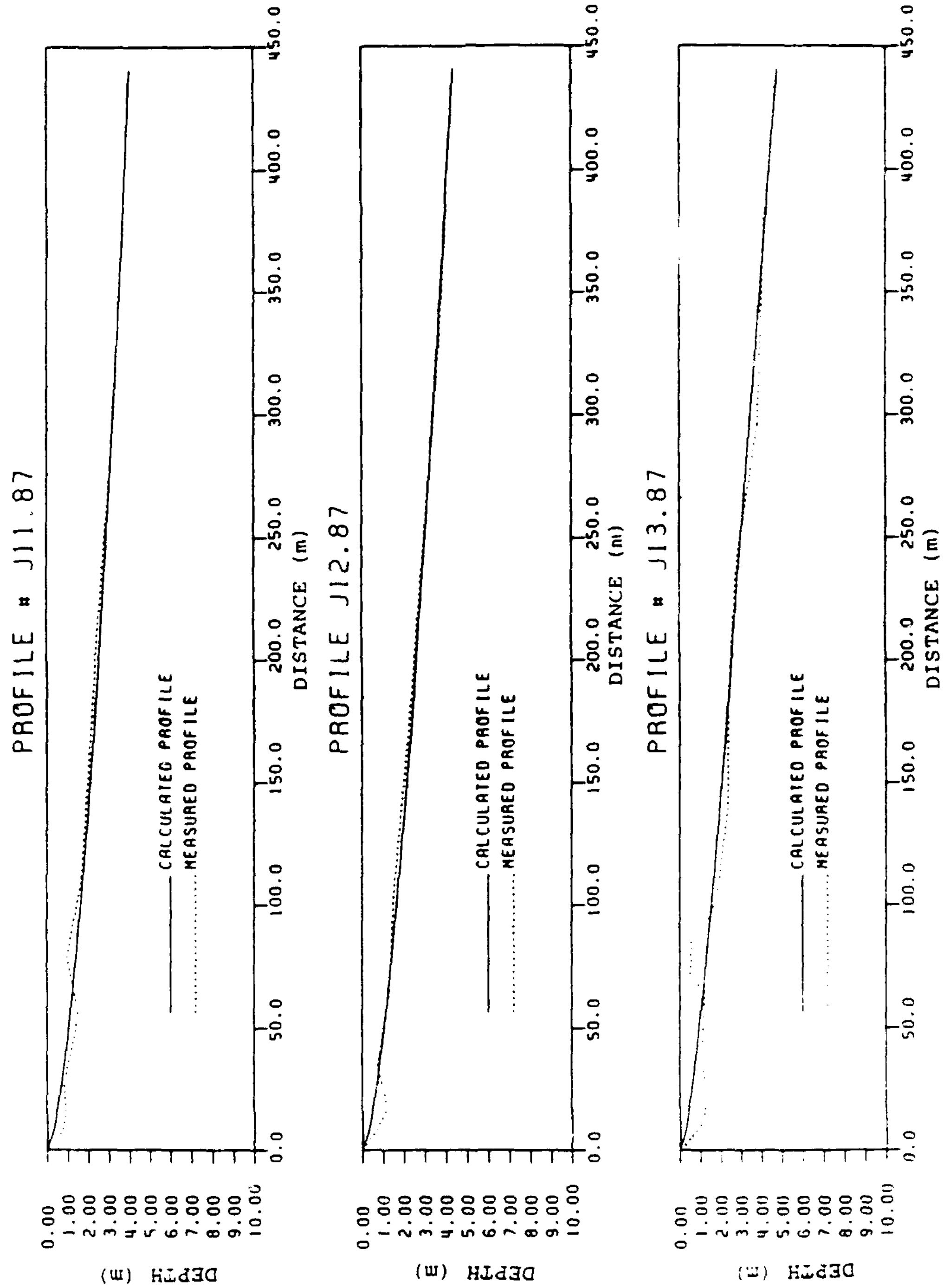


Figure 57. (Sheet 2 of 4)

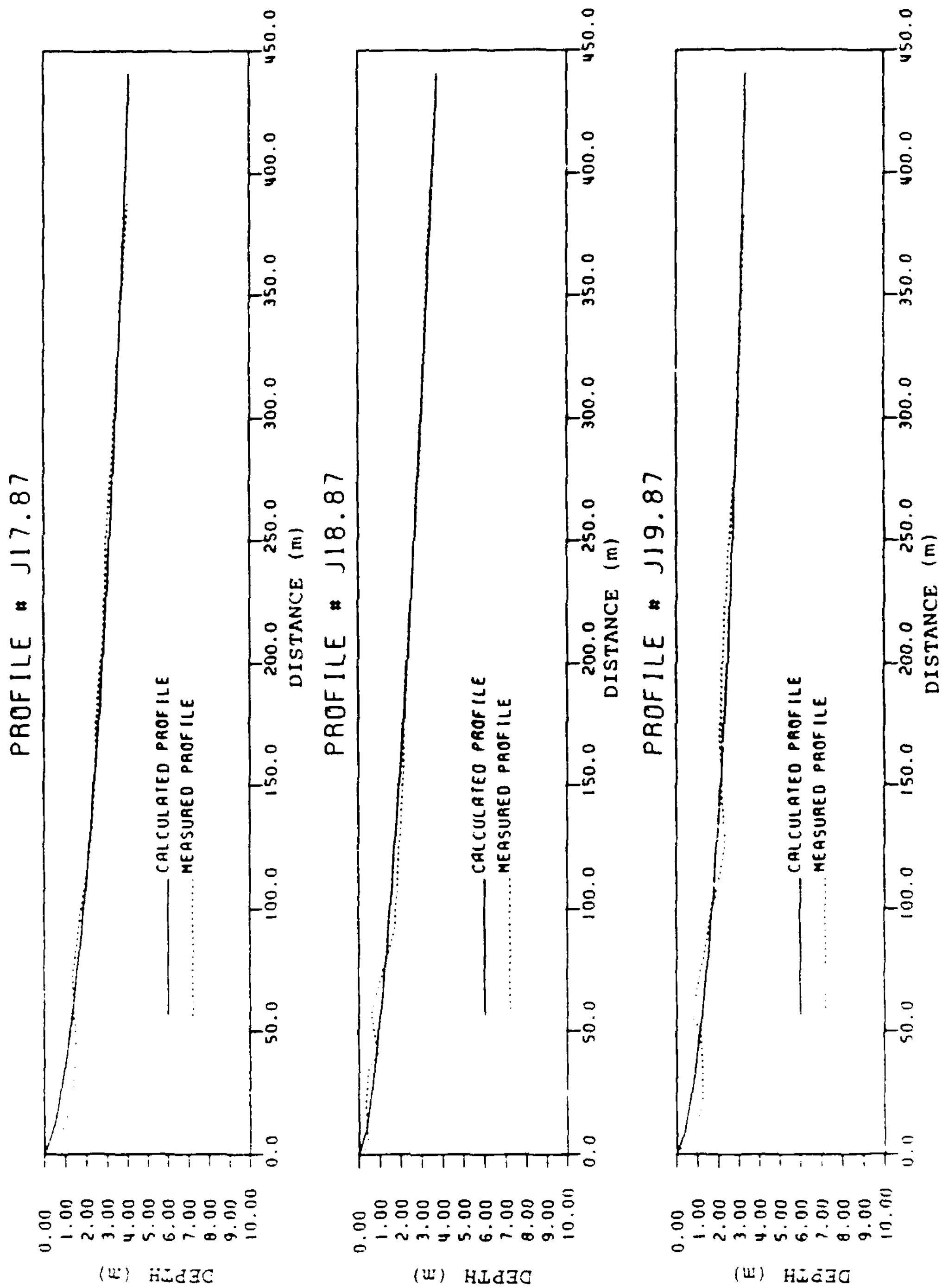


Figure 57. (Sheet 3 of 4)

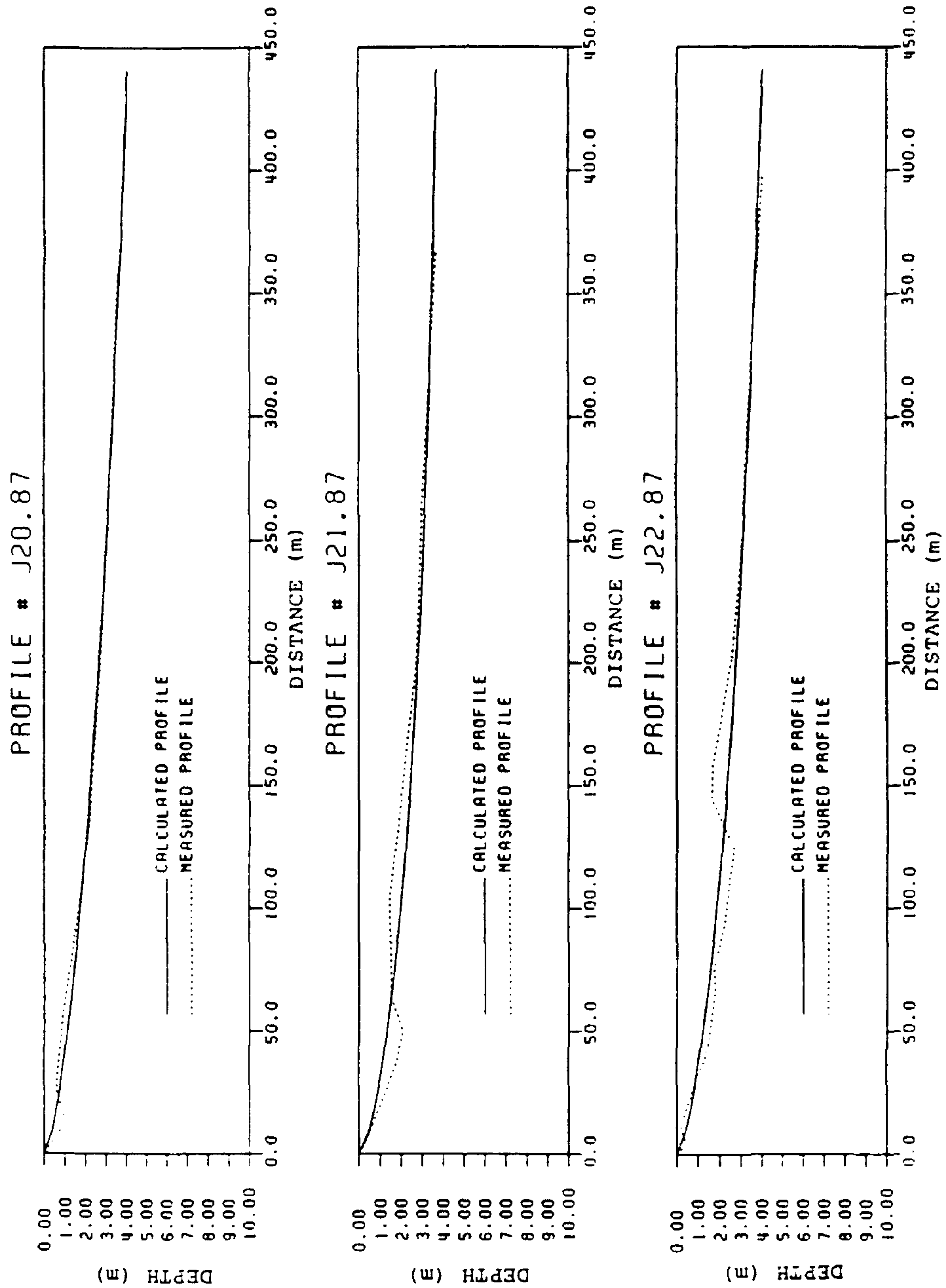


Figure 57. (Sheet 4 of 4)

The second type of analysis presented compares the effective A parameter in the shallower and deeper portions of the profiles before nourishment (1973) and after nourishment (1981 and 1987). Based on the exponential distribution that resulted in the profile fits in Figure 57, the A values near the shore (A_o), at the end of the profile lines, and the average of these two are compared for 1983, 1981, and 1987 in Figures 58, 59, and 60, respectively. In those plots, north is to the right. It is noted that for pre-nourishment conditions (1973), there was a substantial difference between the shallow- water and end-of-line A values. In 1981 (Figure 59), after nourishment, the range had decreased some; however, the mean was about the same. This is interpreted as due to the beach berm being displaced by the relatively finer nourishment sand into deeper water, such that the nearshore and end-of-line A values were more nearly the same. Finally, in 1987 (Figure 60) the A values at the shoreline and end-of-line are nearly the same. This is interpreted as the nourished sediment being transported over the entire profile such that the A parameter is approximately uniform across the profile.

A simpler type of analysis, but similar to that presented in the preceding paragraph, was carried out based on the *overall* characteristic of the profile. In particular, if $h = Ay^{2/3}$ is appropriate, then the average slope $\bar{s}$ to the end of line where the water depth is h' at an offshore distance y' is

$$\bar{s} = \frac{A_s^{3/2}}{(h')^{1/2}} \tag{15}$$

Hence the A value based on overall slope A_s is

$$A_s = [\bar{s}(h')^{1/2}]^{2/3} \tag{16}$$

Similarly, it can be shown readily that the volume per unit length V to the end of the line is

$$V = \frac{3}{5} A_V (y')^{5/3} = \frac{3}{5} \frac{(h')^{5/2}}{(A_V)^{3/2}} \tag{17}$$

Thus the A value based on volume A_V is

$$A_V = \left[\frac{3}{5} \frac{(h')^{5/2}}{V} \right]^{2/3} \tag{18}$$

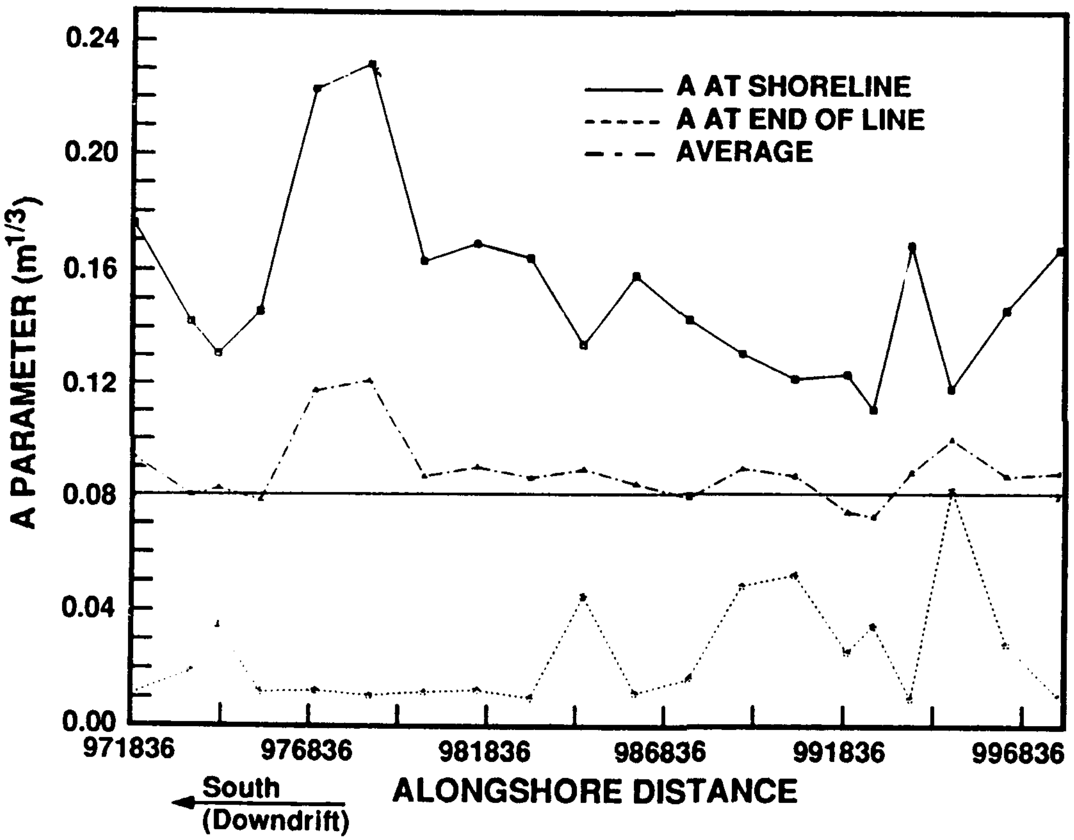


Figure 58. Longshore variations of *A* values at shoreline, end of line, and average, 1973, *A* values based on exponential fit to *A* parameter distribution. Jupiter Island, Florida

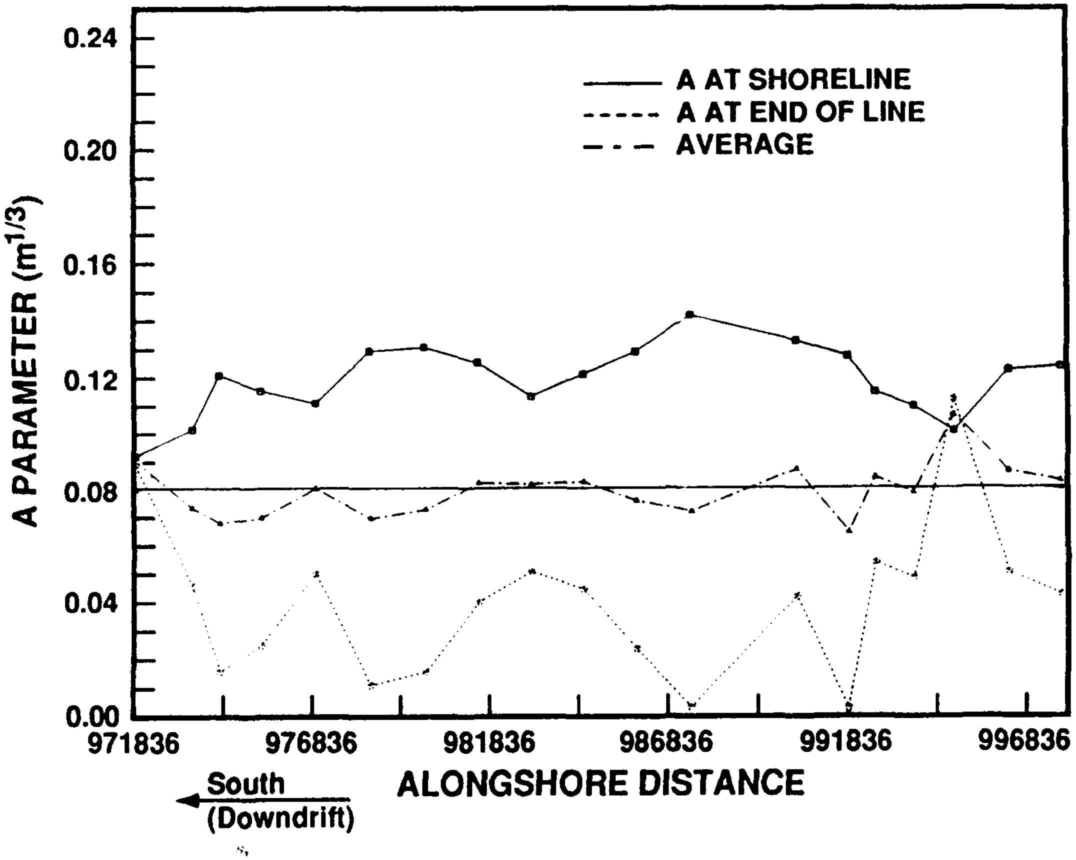


Figure 59. Longshore variations of *A* values at shoreline, end of line, and average, 1981. *A* values based on exponential fit to *A* parameter distribution. Jupiter Island, Florida

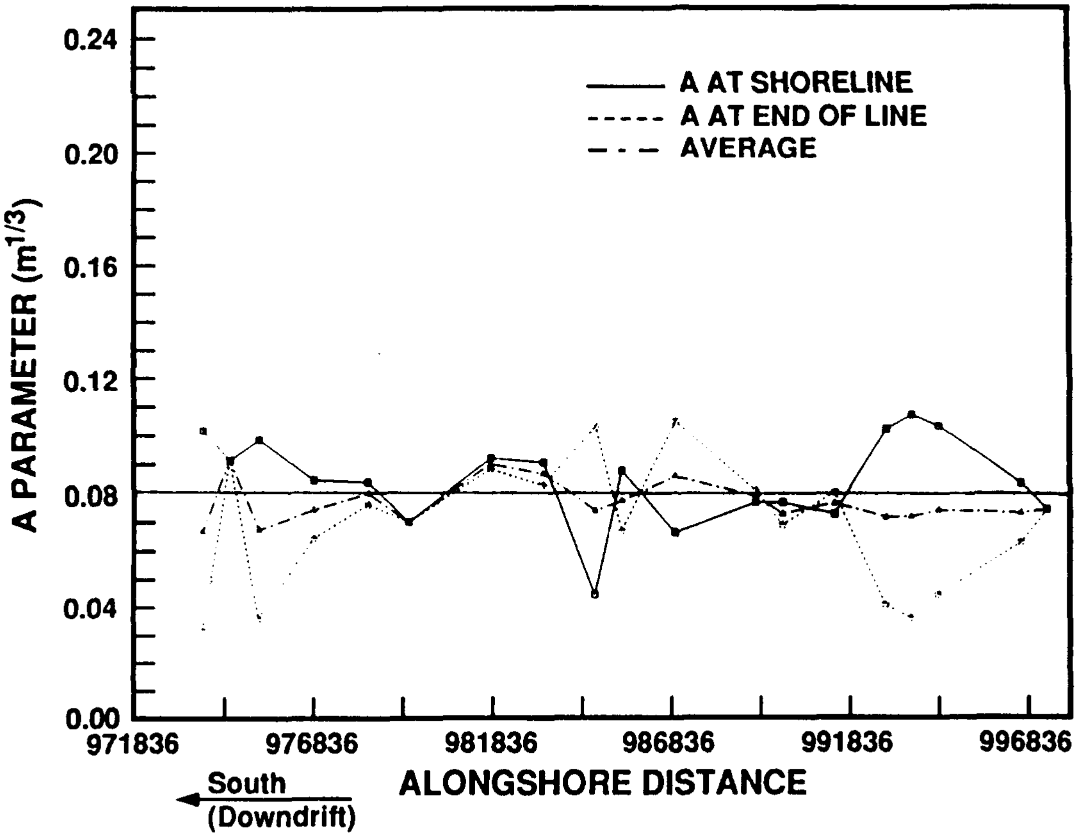


Figure 60. Longshore variations of *A* values at shoreline, end of line, and average, 1987. *A* values based on exponential fit to *A* parameter distribution. Jupiter Island, Florida

In interpreting the differences between A_s and A_V, it is noted that if the sediment is coarser nearshore, the values of A will be greater than offshore, and it is to be expected that $A_V > A_s$, as will be seen. Figures 61, 62, and 63 each present A values based on slopes and volumes, and the average of the two for 1973, 1981, and 1987, respectively. The results and interpretation are generally similar to those presented for Figures 58, 59, and 60. Prior to nourishment (Figure 61), the A values based on volumes were somewhat greater than those based on slopes and the average A value was approximately $0.10\ m^{1/3}$. Following nourishment, the relative difference between the two A values is approximately the same, except that the average A has decreased somewhat. Finally, in 1987, after further nourishment, the A value had decreased further to less than 0.08 $m^{1/3}$ and the A values based on area and volume are nearly the same.

Conclusions Based on Field Data

Based on analysis of available field data from two nourishment sites at Delray Beach, Florida, and Jupiter Island, Florida, the following are concluded. A "blindfolded" comparison of predicted and measured (1988) profiles for Delray Beach shows good agreement (Figure 54). The computed profiles in this comparison are based on an exponential fit to the A values associated with the mean grain sizes. Differences exist primarily for depths greater than 4 to 5 m and may be attributed to the nourished profile equilibrating only to these depths. Additionally, by allowing the parameters in the exponential A representation to be free, a good fit is obtained across the entire profile (Figure 55). At Jupiter Island, since there are no cross-shore sediment size data, the analysis concentrated on the variation of A values in the nearshore and near the end of the line. Two methods were employed and although the general results of the two methods were similar, the quantitative results differed. Based on an exponential A fit, prior to nourishment the shoreline A value was substantially greater than that at the end of the line. By 1987, these two values were nearly the same and the mean value had not changed appreciably from the pre-nourished values. A second approach to examining the A values nearshore and near the end of the line is based on the average slopes and volumes of the measured profiles. Again, prior to nourishment, the two A values differed, whereas in 1987, they were virtually identical. Results using the second method differed from those based on the exponential A fit in that the post-nourishment average A value decreased to approximately 0.08 $m^{1/3}$ from the pre-nourishment value of 0.10 $m^{1/3}$. This result is qualitatively consistent with the use of nourishment sediment that is finer than the native sediment.

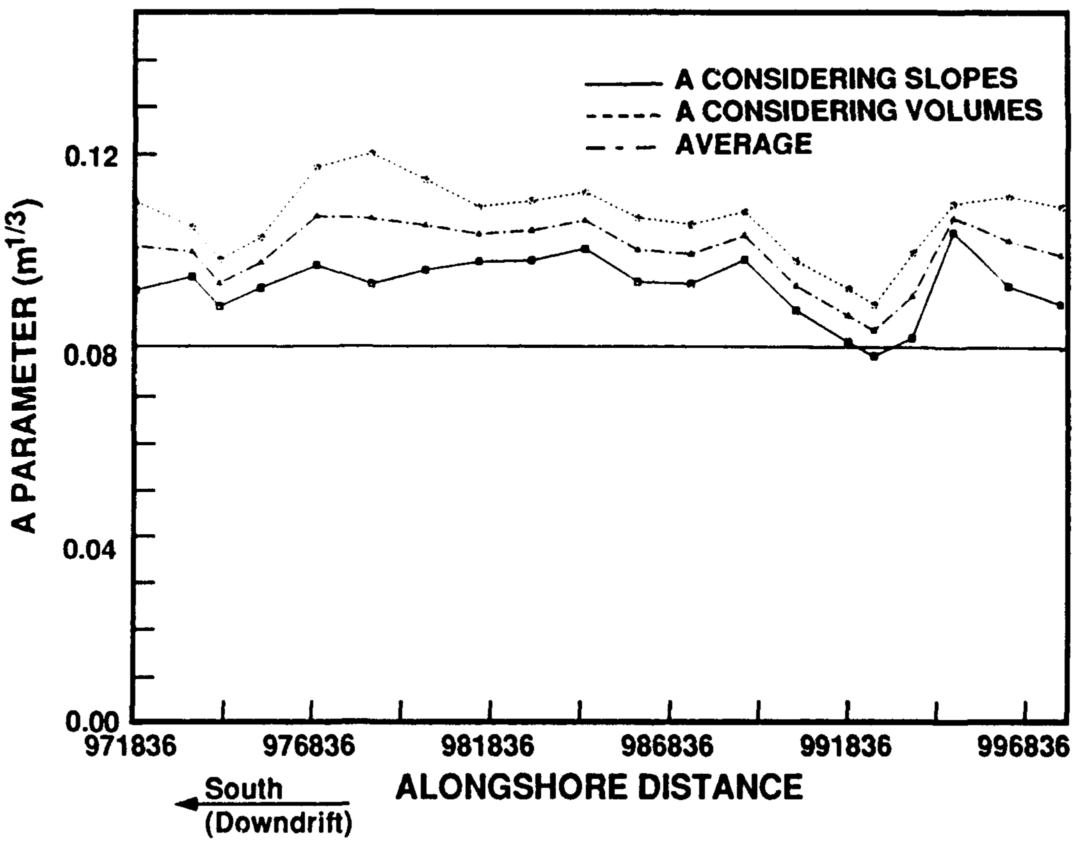


Figure 61. Longshore variations of *A* values based on slopes, volumes, and the average of the two, Jupiter Island, Florida, 1973

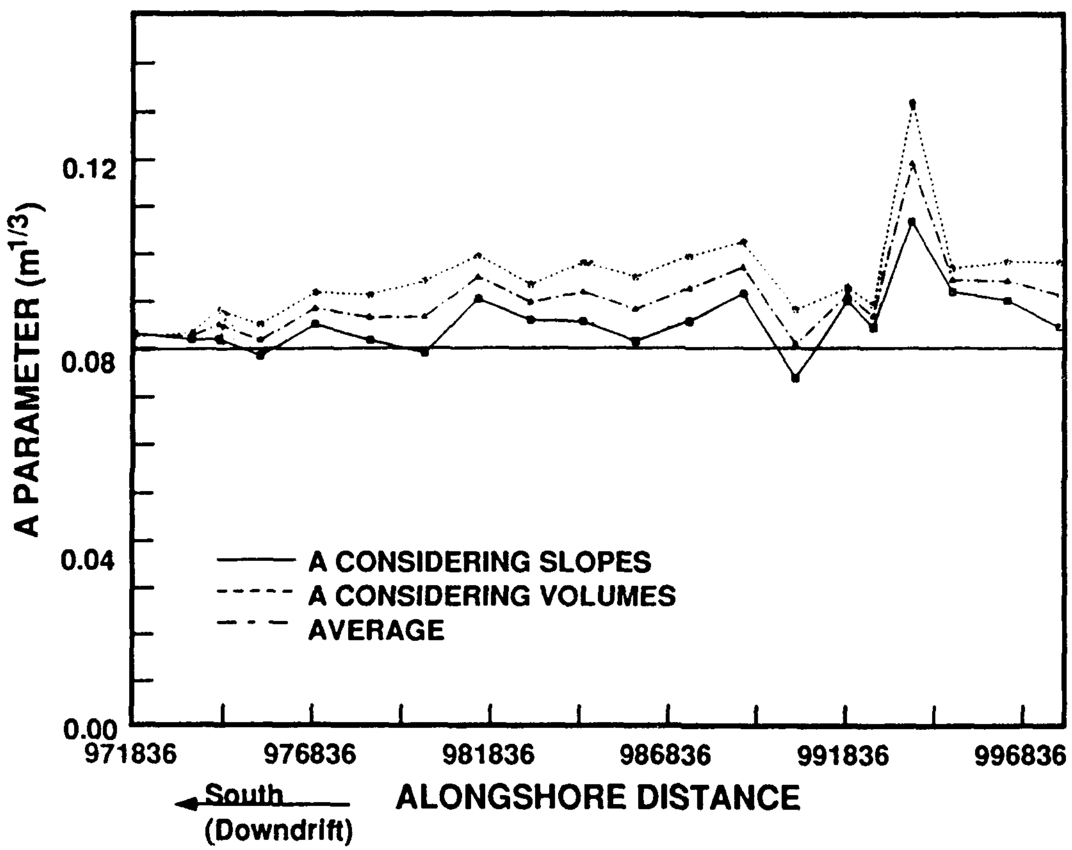


Figure 62. Longshore variations of *A* values based on slopes, volumes, and the average of the two, Jupiter Island, Florida, 1981

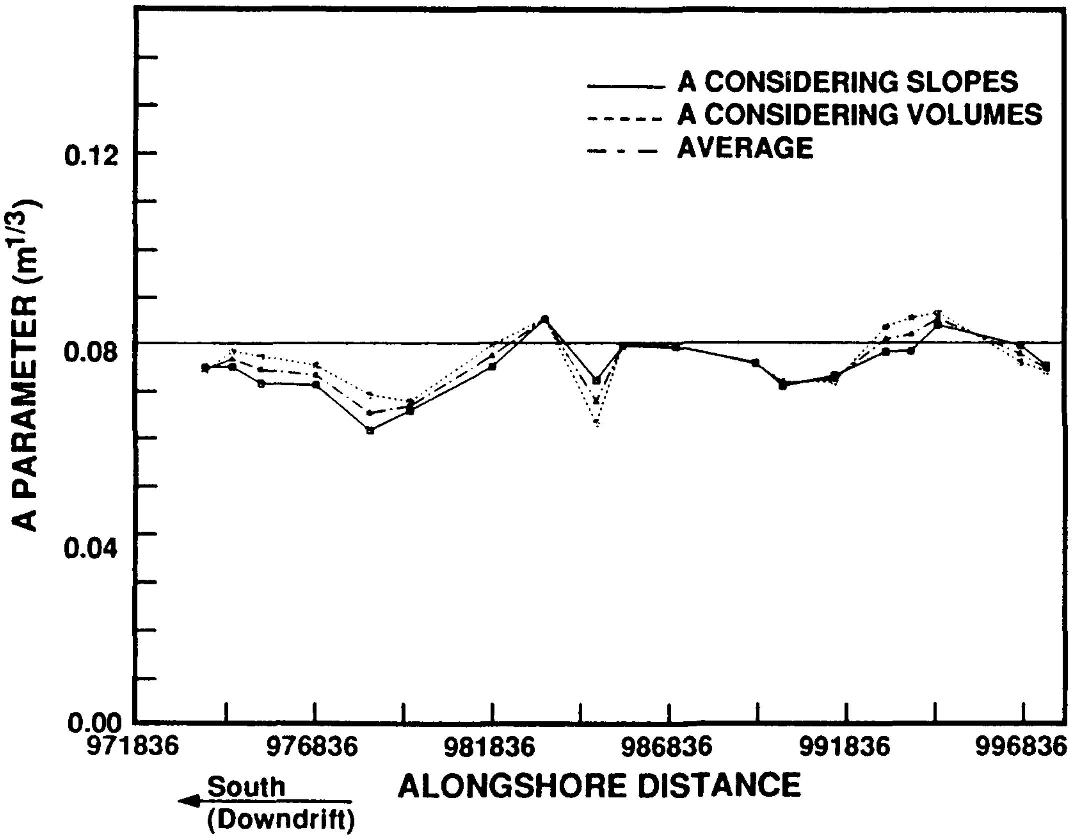


Figure 63. Longshore variations of *A* values based on slopes, volumes, and the average of the two, Jupiter Island, Florida, 1987

6 Summary, Conclusions, and Recommendations for Further Research

Summary

A method has been developed and illustrated by application for the prediction of the two-dimensional beach profile equilibrium resulting from the placement of a specified volume of a well-mixed sand of arbitrary size distribution. It is shown that upon placement of a given volume of material, three types of profiles can result, depending on the material size characteristics, the volume, berm height, and closure depth. These are: (a) intersecting, (b) non-intersecting, and (c) submerged profiles. The method is applicable to the first two types. The method assumes that, locally, the profile is in equilibrium with the profile scale characteristics consistent with a relationship developed by Moore (1982). An iterative method is employed which ensures that the volume eroded from the placed profile is equal to that deposited seaward.

Applications provided to illustrate the method have included idealized grain size distributions and profiles, a specified grain size distribution and profile, and a range of various mean grain sizes and sorting.

Limited small-scale wave tank tests were conducted to investigate sediment sorting occurring due to profile evolution from an initially planar slope. Surveys and surface sand samples were taken across the profile at approximately 0, 1, 5, 10, and 24 hr after commencement of testing. The sand samples were later analyzed for grain size distribution. Although the results were not completely consistent, it was found that in all six cases, the mean sediment size decreased with seaward distance from the equilibrium shoreline.

The concepts of the method were compared where possible with field data from Delray Beach, Florida, and Jupiter Island, Florida, both of which have been nourished on multiple occasions. It was found that the profile shapes

could be predicted reasonably well using the methods employed here. Additionally, the effects of nourishment at Jupiter Island caused changes in profile shape consistent with the concepts employed.

Conclusions

Previous methods for assessing the relative quality of sediment sources for beach nourishment have compared the textural properties of borrow and native material and result in an "overfill factor." However, these methods do not provide a rational capability for predicting the equilibrium dry beach width, an important design quantity and one that can affect significantly the project benefit/cost ratio.

Comparisons of predictions with laboratory-derived profiles were reasonably encouraging, with the predictions overestimating the depths in three experiments, underestimating the depth in two experiments, and generally agreeing in the remaining experiment.

The laboratory cross-shore mean grain sizes seaward of the equilibrium shoreline were generally consistent with those expected and incorporated in the method. However, the data included a considerable distribution of sizes at a given location, whereas the method employed considered the particles at any location to be perfectly sorted.

Blindfolded tests conducted using the Delray Beach, Florida, cross-shore grain size data yielded good comparisons between measured and predicted beach profiles. The agreement was poorer for water depths exceeding 4 to 5 m, beyond which it is believed that the nourished profile either did not extend or did not equilibrate.

The method presented in this report is considered to be one step toward a rational procedure for assessing the complex performance of nourishment projects with realistic sediment characteristics.

Recommendations for Further Research

A great deal remains to be accomplished toward the objective of developing rational procedures for predicting the performance of beach nourishment projects with realistic characteristics. All future beach nourishment projects should be monitored carefully and completely. Of primary importance are: (a) the three-dimensional evolution with time; (b) the forcing functions, i.e., the directional wave spectrum, and, if near an inlet, the currents; and (c) the initial and evolving grain-size characteristics in both the cross-shore and longshore dimensions.

Methods should be developed for predicting the cross-shore distribution of sediment sizes. There is a need to develop procedures for representing the onshore transport and sorting of sediment placed as "profile nourishment," i.e., as an underwater deposit.

Large-scale wave tank tests should be conducted of cross-shore profile and sediment sorting evolution. Finally, research should be directed toward development of a process-based method of predicting cross-shore profile and sediment sorting evolution.

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Appendix A
Listing of Program EQPR.FOR and Input and Output Files for Examples 1 and 4

```
C         THIS PROGRAM EQPR.FOR
C  ***********************************************************************
C  * THIS PROGRAM DEVELOPED FOR THE COASTAL ENGINEERING RESEARCH CENTER *
C  *  AND  CALCULATES THE EQUILIBRIUM BEACH PROFILE FOR A NOURISHMENT   *
C  *  SAND OF SPECIFIED SIZE CHARACTERISTICS  October 23,1991 (Revised) *
C  ***********************************************************************
C
      COMMON/A/ Y(800),HO(800),HP(800),HEQ(800),DC(800),AC(800),
     1         PCC(800),VUSEDC(800),P(40),D(40),A(40),DI(40),AI(40)
      DIMENSION WORD(20),VGENV(50),VUSEDV(50),PSV(50),DYEQV(50),
     1           IFLAG(6)
      OPEN(UNIT=5,FILE='EQPR.INP',STATUS='OLD')
      OPEN(UNIT=6,FILE='EQPR.OUT',STATUS='NEW')
  122 FORMAT(20A4)
  123 FORMAT(6F8.2,2I6)
  124 FORMAT(8F8.2)
  126 FORMAT(//,'  STARTING LOOP ITS = ',I6,'  DYEQ = ',F8.3)
  160 FORMAT(8I6)
  162 FORMAT(F8.2,3I6,2F8.2)
  164 FORMAT(8I6)
  166 FORMAT(I6,3F8.2)
  167 FORMAT(I6,4F10.3)
  168 FORMAT('  IMAX IFLAG2 IEND  HEND     DYEQO      DYEQ    VUSED',
     1 5X,'  VGEN  HCROSS')
  170 FORMAT(3(I6,2F8.2))
  178 FORMAT('     I      Y(I)     HO(I)    HP(I)     HEQ(I)     AC(I)',1X,
     1'  VUSEDC(I)   PCC(I)')
  180 FORMAT(I6,7F9.3)
  181 FORMAT('  ITS = ',I3,'  DYEQ = ',F8.4,' PSV = ',F8.3,'  VUSED = '
     1,E10.4, ' VGEN = ',E10.4,' HO = ',F8.3,' HP = ',F8.3,' HEQ = ',
     2 F8.3)
  182 FORMAT('  SOLUTION REACHED,  NON-INTERSECTING PROFILES')
  183 FORMAT(' SOLUTION REACHED,  INTERSECTING PROFILES')
  184 FORMAT(//)
  185 FORMAT('      I    DYEQ(I)   VGENV(I)   VUSED(I)',
     1,'   PSV(I)')
C ************************************************************************
C        NOTE: ALL DIMENSIONS ARE IN METERS                              *
C              HO(I)=ORIGINAL PROFILE                                    *
C              HP(I)=PLACED (NOURISHED) PROFILE                          *
C              HEQ(I)=EQUILIBRIUM PROFILE                                *
C ************************************************************************
C     ESTABLISH INITIAL AND PLACED PROFILES
C
      READ(5,122)(WORD(I),I=1,20)
      WRITE(6,122)(WORD(I),I=1,20)
      WRITE(*,122)(WORD(I),I=1,20)
      CALL PROCHAR(JPTYPE,HO,HP,SP,VADD,IMAX,HBERM,DY,Y,DYO,HSTAR,
     1 SEQ,HMIX)
C
C     READ IN CHARACTERISTICS OF PLACED SAND
C
      CALL SEDCHAR(LMAX,LIMAX,D,P,DI,AI)
C         SUBROUTINE ATRANS TRANSFORMS SEDIMENT DIAMETERS TO A VALUES
      CALL ATRANS(LMAX,LIMAX,D,P,A,DI,AI)
C        OUTER HSTAR LOOP
C
      READ(5,167)ITSMAX,DYEQ1
      DDYEQ=DYEQ1/20.0
```

Figure A1. Listing for program EQPR.FOR (Sheet 1 of 10)

```
      IF(DDYEQ.LT.1.0)DDYEQ=1.0
      ITERMX=3
      PS=0.7
      VAVAIL=PS*VADD
      DYEQ=DYEQ1
C        THIS LOOP SOLVES FOR PS FOR TRIAL DYEQ
      DO 200 ITS=1,ITSMAX
      VGEN=VAVAIL
      VUSED=VAVAIL
      DYEQO=DYEQ
      DO 198 ITER=1,ITERMX
      DO 196 I=1,800
      AC(I)=0.0
      VUSEDC(I)=0.0
      PCC(I)=0.0
  196 HEQ(I)=0.0
      VAVAIL=VGEN
      CALL PSSR(DYEQ,DY,HBERM,VAVAIL,VUSED,
     1          SEQ,IMAX,LMAX,HMIX,HSTAR,VGEN,ITS,ITSMAX,
     2          IM,IFLAG)
C     WRITE(*,*)ITS,IPS,DYEQ,PS,VUSED,VGEN
  198 CONTINUE
      DYEQV(ITS)=DYEQ
      VGENV(ITS)=VGEN
      VUSEDV(ITS)=VUSED
      PSV(ITS)=VGEN/VADD
      PSCUR=PSV(ITS)
      VAVAIL=PSCUR*VADD
      WRITE(*,181)ITS,DYEQ,PSV(ITS),VUSED,VGEN,HO(IM),HP(IM),HEQ(IM)
      IF(ITS.GT.1)GO TO 199
      AA=VGEN-VUSED
      DDYEQ=DDYEQ*AA/ABS(AA)
      WRITE(*,*)ITS,DYEQ,DDYEQ,AA
      GO TO 200
  199 CONTINUE
      AAC=VGENV(ITS)-VUSEDV(ITS)
      AAP=VGENV(ITS-1)-VUSEDV(ITS-1)
      IF(AAC.EQ.0.0)GO TO 202
      IF(AAC/AAP.LT.0.0)DDYEQ=-DDYEQ/2.0
  200 DYEQ=DYEQ+DDYEQ
  202 CONTINUE
      WRITE(*,*)(I,IFLAG(I),I=1,6)
  400 CONTINUE
  800 CONTINUE
      WRITE(6,184)
      WRITE(6,178)
      WRITE(6,180)(I,Y(I),HO(I),HP(I),HEQ(I),AC(I),VUSEDC(I),PCC(I),
     1   I=1,IMAX)
      WRITE(6,184)
      WRITE(6,185)
      WRITE(6,167)(I,DYEQV(I),VGENV(I),VUSEDV(I),PSV(I),I=1,ITSMAX)
      WRITE(6,184)
      IF(IFLAG(5).EQ.1)WRITE(*,182)
      IF(IFLAG(6).EQ.1)WRITE(*,183)
      IF(IFLAG(5).EQ.1)WRITE(6,182)
      IF(IFLAG(6).EQ.1)WRITE(6,183)
      CLOSE(UNIT=5)
      CLOSE(UNIT=6)
      STOP
      END
```

Figure A1. (Sheet 2 of 10)

```
C
C        THIS SUBROUTINE CALCULATES THE NEW EQUILIBRIUM PROFILE AND
C        DETERMINES THE TYPE OF PROFILE (INTERSECTING OR NON-INTER)
C     -------------------------------------------------------
      SUBROUTINE PSSR(DYEQ,DY,HBERM,VAVAIL,VUSED,
     1          SEQ,IMAX,LMAX,HMIX,HSTAR,VGEN,
     2          ITS,ITSMAX,IM,IFLAG)
C     -------------------------------------------------------
      COMMON/A/ Y(800),HO(800),HP(800),HEQ(800),DC(800),AC(800),
     1 PCC(800),VUSEDC(800),P(40),D(40),A(40),DI(40),AI(40)
      DIMENSION HDP(5),HDC(5),IFLAG(6)
   20 FORMAT(' OPUT SR PSSR ,ITS=',I4,' ITSMAX=',I4,
     1  ' DYEQ=',F8.3,' VUSED=',F8.3,' VGEN=',F8.3)
   22 FORMAT(3I5,7F10.3)
   23 FORMAT(3I5,12F10.3)
   24 FORMAT('   REACHED CROSS-OVER POINT')
  160 FORMAT(8F8.2)
      I1=DYEQ/DY+2
      DHP=0.0
C       IFLAG(1)=1 MEANS HP(I+1)>HEQ(I+1)
      IFLAG(1)=0
C       IFLAG(2)=1 MEANS INTERSECTING PROFILES
      IFLAG(2)=0
C       IFLAG(3)=1 MEANS PROFILE HAS REACHED CRITERION FOR END OF
C       INTERSECTING OR NON-INTERSECTING PROFILE
      IFLAG(3)=0
C       IFLAG(4)=1 MEANS SEAWARD OF PLACED PROFILE
      IFLAG(4)=0
C       IFLAG(5)=1 MEANS PROFILE HAS REACHED CLOSURE DEPTH
      IFLAG(5)=0
C       IFLAG(6)=1 MEANS INTERSECTION OF ORIGINAL AND EQUILIBRIUM PROFILES
      IFLAG(6)=0
      HEQ(I1)=HBERM
      HMIXP=HMIX
      HCROSS=0.0
      IMM1=IMAX-1
      DYA=Y(I1)-DYEQ
      VGEN=(DYA+HMIX/SEQ)*(DYA*SEQ+HMIX)/2.0
      VUSED=VGEN-DYA**2*SEQ/2.0
      WRITE(*,*)I1,DYA,SEQ,HMIX,VGEN,VUSED
      HDC(1)=HP(I1)
      HDC(2)=HEQ(I1)
      HDC(3)=HEQ(I1)+HMIX
      HDC(4)=HO(I1)
      YC=Y(I1)
      DO 100 I=I1,IMM1
      IM=I+1
      DO 96 L=1,4
   96 HDP(L)=HDC(L)
      HMIXP=HMIXC
      DYY=DY
      YP=YC
      IEND=I+1
      HMIXC=HMIX
      YC=Y(I+1)
C         FOLLOWING SR CALCULATES NEW EQUIL PROFILE
      CALL YNEW(I,Y,HEQ,VUSED,SEQ,VAVAIL,A,P,LMAX,
     1          DYEQ,HBERM,AC,VUSEDC,PCC)
C     WRITE(6,*)I,HEQ(I+1),HO(I+1)
      HEND=HEQ(I+1)
```

Figure A1. (Sheet 3 of 10)

```
C     IF(ITS.NE.8)GO TO 36
C     WRITE(6,23)I,IFLAG(1),IFLAG(4),Y(I+1),HEQ(I+1),HO(I+1),HP(I+1),
C    1           VGEN,VUSED,VAVAIL,DHP,DYY,YP
   36 DH=HO(I+1)-HEQ(I+1)
      IF(DH.LT.HMIX.AND.IFLAG(1).EQ.0)HMIXC=DH
      HDC(1)=HP(I+1)
      HDC(2)=HEQ(I+1)
      HDC(3)=HEQ(I+1)+HMIXC
      HDC(4)=HO(I+1)
      IA=2
      IB=3
      IF(IFLAG(1).EQ.0)GO TO 40
      IB=1
      IF(IFLAG(4).EQ.0)GO TO 40
      IB=4
      IF(IFLAG(5).EQ.1.OR.IFLAG(6).EQ.1)GO TO 200
   40 CALL VOL(HDP,HDC,IA,IB,YP,YC,SEQ,HMIXP,VGEN,VUSED,IFLAG,HSTAR)
  100 CONTINUE
      WRITE(*,20)ITS,ITSMAX,DYEQ,VUSED,VGEN
  200 RETURN
      END
C     ------------------------------------
      SUBROUTINE PROCAL(Y,I,AO,SO,HO,HBERM)
C     ------------------------------------
      DIMENSION Y(800),HO(800)
   20 FORMAT(2I6,8F8.3)
      IF(Y(I).GE.-HBERM/SO) GO TO 40
      HO(I)=HBERM+(Y(I)-Y(1))*SO
      GO TO 42
   40 HO(I)=0.2
      DO 80 K=1,20
C     WRITE(*,20)I,K,Y(I),HO(I),SO,AO,HBERM
      EPS=Y(I)+HBERM/SO-HO(I)/SO-(HO(I)/AO)**1.5
      DEPSDH=-1.0/SO-1.5*HO(I)**0.5/AO**1.5
   80 HO(I)=HO(I)-EPS/DEPSDH
   42 RETURN
      END
C
C        THIS SUBROUTINE CALLED FOR EACH Y VALUE TO DETERMINE
C        HEQ(I+1)
C     ---------------------------------------------------
      SUBROUTINE YNEW(I,Y,HEQ,VUSED,SEQ,VAVAIL,A,P,LMAX,
     1                DYEQ,HBERM,AC,VUSEDC,PCC)
C     ---------------------------------------------------
      DIMENSION Y(800),HEQ(800),A(40),P(40),AC(800),VUSEDC(800),
     1          PCC(800)
   20 FORMAT(2I6,6F8.3)
   22 FORMAT('   OUTPUT FROM SR YNEW')
C     NOTE HBERM IS NEGATIVE
C     WRITE(*,22)
      YZERO=-HBERM/SEQ+DYEQ
      IF(Y(I+1).LE.YZERO)GO TO 40
C        THIS SECTION FOR PORTION OF PROFILE BELOW WATER
      DYY=Y(I+1)-Y(I)
      HPREV=HEQ(I)
      DYY2=Y(I+1)-YZERO
      IF(DYY2.GE.DYY)GO TO 34
C         THIS SECTION FOR FIRST BELOW WATER POINT
      DYY=DYY2
      HPREV=0.0
```

Figure A1. (Sheet 4 of 10)

```
   34 PC=VUSED/VAVAIL
      VUSEDC(I)=VUSED
      IF(PC.GT.1.0)PC=1.0
      PCC(I)=PC
      IF(PC.GT.1.0)PC=1.0
C       THIS SUBROUTINE DETERMINES CURRENT A PARAMETER VALUE
      PC=1.0-PC
      CALL INTERP(P,A,PC,AB,LMAX)
      AC(I)=AB
C     WRITE(6,20)I,LMAX,VAVAIL,VUSED,VGEN,AB
      HEQ(I+1)=HPREV+0.02
      DO 36 LL=1,10
      HBAR=0.5*(HPREV+HEQ(I+1))
      BB=1.0/SEQ+1.5*SQRT(HBAR)/AC(I)**1.5
      DHDY=1.0/BB
      HEQ(I+1)=HPREV+DHDY*DYY
   36 CONTINUE
      GO TO 102
C       NEXT STATEMENT FOR PORTION OF PROFILE ABOVE WATER
   40 HEQ(I+1)=HBERM+(Y(I+1)-DYEQ)*SEQ
  102 RETURN
      END
C     -------------------------------
      SUBROUTINE INTERP(X,Y,XC,YC,NAS)
C     -------------------------------
      DIMENSION X(40),Y(40)
   22 FORMAT('   OUTPUT FROM INTERP')
   24 FORMAT(2I6,6F8.3)
   26 FORMAT('  *******',2I6,6F8.3)
C     WRITE(*,22)
      NASM1=NAS-1
      DO 40 N=1,NASM1
       NC=N
      IF(XC.GE.X(N).AND.XC.LE.X(N+1).AND.X(N+1).GT.X(N)) GO TO 42
      IF(XC.LE.X(N).AND.XC.GE.X(N+1).AND.X(N+1).LT.X(N)) GO TO 42
   40 CONTINUE
   42 DX=X(NC+1)-X(NC)
      DY=Y(NC+1)-Y(NC)
      YC=Y(NC)+(XC-X(NC))*DY/DX
C     WRITE(8,26)NC,NAS,XC,YC,DY,DX,X(NC),Y(NC)
C     WRITE(*,22)
C     WRITE(*,24)NC,NAS,XC,YC
      RETURN
      END
C
C      THIS SUBROUTINE ESTABLISHES THE CHARACTERISTICS OF THE
C      ORIGINAL AND PLACED PROFILES
C     -----------------------------------------------------
      SUBROUTINE PROCHAR(JPTYPE,HO,HP,SP,VADD,IMAX,HBERM,
     1                   DY,Y,DYO,HSTAR,SEQ,HMIX)
C     -----------------------------------------------------
      DIMENSION Y(800),HO(800),YV(100),HVO(100),HP(800)
   20 FORMAT(2I6,9F7.2)
   22 FORMAT('    ORIGINAL PROFILE')
   24 FORMAT('    PLACED PROFILE')
   26 FORMAT(3(I6,2F8.2))
   28 FORMAT('  ITERATION=',I6,'IL='I6,' VADDED=',F8.2,'    VUSED=',F8.2,
     1 2X,'DYO=',F8.3)
   32 FORMAT('   VADDED = ',F8.2,'   VUSED = ',F8.3,'  DYO = ',F8.3)
   31 FORMAT(6I6)
```

Figure A1. (Sheet 5 of 10)

```
   30 FORMAT('CHECK',I6,8F8.3)
   33 FORMAT(8F8.2)
      READ(5,20)JPTYPE,IMAX,DY,AO,SO,SP,VADD,SEQ,HBERM,HSTAR,HMIX
      WRITE(*,20)JPTYPE,IMAX,DY,AO,SO,SP,VADD,SEQ,HBERM,HSTAR,HMIX
      Y(1)=0.0
      DO 10 I=2,IMAX
   10 Y(I)=Y(I-1)+DY
C        JTYPE=1,READ IN HO(ORIGINAL DEPTHS); JTYPE=0,CALCULATE
C        DEPTHS
      IF(JPTYPE.EQ.1) GO TO 42
      DO 40 I=1,IMAX
   40 CALL PROCAL(Y,I,AO,SO,HO,HBERM)
      GO TO 44
   42 READ(5,31)IMAXP
      READ(5,33)(YV(I),HVO(I),I=1,IMAXP)
      HO(1)=HVO(1)
      DO 43 I=2,IMAX
      XC=Y(I)
      CALL INTERP(YV,HVO,XC,YC,IMAXP)
   43 HO(I)=YC
   44 CONTINUE
C     WRITE(6,22)
C     WRITE(6,26)(I,Y(I),HO(I),I=1,IMAX)
      GO TO 60
C       THIS SECTION ESTABLISHES PLACED PROFILE
C  60 WRITE(6,20)IMAX,JPTYPE,VADD
   60 CONTINUE
      DYO=20.0
      DO 80 K=1,20
      DO 50 I=1,IMAX
   50 HP(I)=0.0
      VUSED=0.0
C     WRITE(6,20)IMAX,JPTYPE,VADD
      HP(1)=HBERM
      DO 78 I=2,IMAX
      IM=I
      AB=0.0
      IL=I
      IF(Y(I).LE.DYO)HP(I)=HBERM
      IF(Y(I).GT.DYO)HP(I)=HBERM+(Y(I)-DYO)*SP
      DHC=HO(I)-HP(I)
      DYY=DY
      AC=HO(I-1)-HP(I-1)
      IF(HP(I).LT.HO(I)) GO TO 64
      AA=Y(I)-Y(I-1)
      AB=HO(I)-HP(I)
      AC=HO(I-1)-HP(I-1)
      DYY= AC*DY/(AC-AB)
      DHC=0.0
   64 VUSED=VUSED+0.5*DYY*(DHC+AC)
C     WRITE(6,20)K,I,VADD,VUSED,AA,AB,AC,DYY
      IF(AB.LT.0.0)GO TO 79
   78 CONTINUE
   79 DYO=DYO+(VADD-VUSED)/(HO(IL)-HBERM)
C     WRITE(6,30)IL,DYO,VADD,VUSED,HO(IL),HBERM
      WRITE(*,28)K,IL,VADD,VUSED,DYO
   80 CONTINUE
C     DO 82 I=IM,IMAX
C  82 HP(I)=HO(I)
C     WRITE(6,24)
```

Figure A1. (Sheet 6 of 10)

```
C     WRITE(6,26)(I,Y(I),HP(I),I=1,IMAX)
      RETURN
      END
C     -----------------------------------------------
      SUBROUTINE SEDCHAR(LMAX,LIMAX,D,P,DI,AI)
C     -----------------------------------------------
      DIMENSION D(40),P(40),DI(40),AI(40),PD(40)
   20 FORMAT(I6)
   21 FORMAT(2I6,6F8.2)
   22 FORMAT(8F8.3)
   24 FORMAT('  OUTPUT FROM SUBROUTINE SEDCHAR, LMAX = ',I6)
   26 FORMAT('  INPUT PAIRS OF DI,AI ')
   28 FORMAT('  INPUT PAIRS OF D(mm),P')
   29 FORMAT(I6,4F8.3)
   30 FORMAT(//)
      READ(5,21)JSED,LMAX,XMU,SIG
C     WRITE(6,21)JSED,LMAX,XMU,SIG
      LIMAX=19
      LMM1=LMAX-1
      AA=0.3989
      READ(5,22)(DI(L),AI(L),L=1,LIMAX)
      WRITE(6,30)
      WRITE(6,26)
      WRITE(6,22)(DI(L),AI(L),L=1,LIMAX)
      IF(JSED.EQ.1)GO TO  8
      DO 10 L=1,LMAX
      XL=L
      DD=0.6*SIG
      D(L)=XMU-3.0*SIG+(XL-1)*DD
      BB=((D(L)-XMU)/SIG)**2/2.0
      PD(L)=AA/SIG*EXP(-BB)
      DC=D(L)
   10 D(L)=2.0**(-DC)
C     WRITE(6,22)(D(L),PD(L),L=1,11)
      P(1)=1.0
      P(11)=0.0
      DO 12 L=2,LMM1
   12 P(L)=P(L-1)-(PD(L-1)+PD(L))*DD/2.0
      GO TO 80
    8 READ(5,22)(D(L),P(L),L=1,LMAX)
C     WRITE(6,26)
   80 WRITE(6,30)
      WRITE(6,28)
      WRITE(6,22)(D(L),P(L),L=1,LMAX)
      RETURN
      END
C
C     ------------------------------------------
      SUBROUTINE ADET(I,A,P,AC,VUSED,VAVAIL,LMAX)
C     ------------------------------------------
      DIMENSION A(40),P(40),AC(800)
      PC=VUSED/VAVAIL
      DO 20 L=2,LMAX
      LC=LMAX-L+1
      IF(PC.GT.P(LC).OR.PC.LT.P(LC+1)) GO TO 20
      DP=P(LC)-P(LC+1)
      A1=PC-P(LC+1)
      DA=A(LC)-A(LC+1)
      AC(I)=A(LC+1)-DP*DA/DP
      GO TO 22
```

Figure A1. (Sheet 7 of 10)

```
   20 CONTINUE
   22 CONTINUE
      RETURN
      END
C
C       THIS SUBROUTINE TRANSFERS SEDIMENT DIAMETER ,D, TO
C       SEDIMENT SCALE FACTOR,A
C     -------------------------------------
      SUBROUTINE ATRANS(LMAX,LIMAX,D,P,A,DI,AI)
C     -------------------------------------
      DIMENSION D(40),P(40),A(40),DI(40),AI(40)
   22 FORMAT('   OUTPUT FROM SR ATRANS')
   24 FORMAT(8F8.3)
   26 FORMAT(I6,3F8.3)
   28 FORMAT(2I6,5F8.3)
C     WRITE(6,28)LMAX
      WRITE(*,22)
      DO 20 L=1,LMAX
      DC=D(L)
C     WRITE(6,28)L,LMAX,DC
      CALL INTERP(DI,AI,DC,AC,LIMAX)
C     WRITE(6,26)L,DC,AC,P(L)
   20 A(L)=AC
      WRITE(*,22)
C     WRITE(6,22)
C     WRITE(6,24)(A(L),P(L),L=1,LMAX)
      RETURN
      END
C
C
C      ------------------------------------------------------
      SUBROUTINE VOL(HDP,HDC,IA,IB,YP,YC,SEQ,HMIXP,
     1               VGEN,VUSED,IFLAG,HSTAR)
C      ------------------------------------------------------
      DIMENSION HDP(5),HDC(5),IFLAG(6)
   20 FORMAT('  IFLAG(4) = ',I6)
   22 FORMAT(2I6,7F8.3)
   24 FORMAT('    IFLAG(1) = ',I6)
   26 FORMAT('    IFLAG(5) =',I6,'  IFLAG(6) =',I6)
      HMIX=HMIXP
      IF(IA.EQ.2.AND.IB.EQ.3)GO TO 62
      IF(IA.EQ.2.AND.IB.EQ.1)GO TO 64
      IF(IA.EQ.2.AND.IB.EQ.4)GO TO 66
C      THIS SECTION FOR GENERATION OF PRIMARY VOLUME
   62 IF(HDP(1).LT.HDP(2).AND.HDC(1).GT.HDC(2))IFLAG(1)=1
      IF(IFLAG(1).EQ.0)GO TO 300
C       IF ABOVE CONDITION IS MET, HAVE REACHED CROSS-OVER POINT
      IA=1
      IB=2
      CALL CROSS(HDP,HDC,IA,IB,YP,YC,YINT,HINT)
      DVMIX=(HDP(3)-HDP(2))*(YINT-YP)+HMIX**2/(2.0*SEQ)
      DVGEN=(HDP(2)-HDP(1))*(YINT-YP)/2.0
      VUSED=VUSED+DVMIX+(HDC(1)-HDC(2))*(YC-YINT)/2.0
      VGEN=VGEN+DVGEN+DVMIX
C     IF(HDC(1).EQ.0.0)IFLAG(4)=1
C     WRITE(6,24)IFLAG(1)
C     WRITE(6,22)IA,IB,DVMIX,DVGEN,VUSED,VGEN,YINT,HMIX,SEQ
      GO TO 400
C         TO HERE IF PAST CROSS-OVER POINT BUT BEFORE PROFILE INTERSECTION
   64 IF(HDP(1).LT.HDP(4).AND.HDC(1).GT.HDC(4))IFLAG(4)=1
```

Figure A1. (Sheet 8 of 10)

```
      IF(HDC(1).EQ.0)IFLAG(4)=1
      IF(IFLAG(4).EQ.0)GO TO 300
C     WRITE(6,20)IFLAG(4)
C     IA=4
      CALL CROSS(HDP,HDC,IA,IB,YP,YC,YINT,HINT)
      VUSED=VUSED+(HDP(1)-HDP(2)+HDC(1)-HDC(2))*(YC-YP)/2.0
     1      -(HDC(1)-HDC(4))*(YC-YINT)/2.0
C     WRITE(6,22)IA,IB,VUSED,YC,YP,YINT,HDP(1),HDP(2),HDC(1)
C     WRITE(6,22)IA,IB,HDC(2),HDC(4)
      GO TO 400
   66 IF(HDP(4).GT.HDP(2).AND.HDC(4).LT.HDC(2))IFLAG(6)=1
      IF(HDC(2).GT.HSTAR)IFLAG(5)=1
      IF(IFLAG(5).NE.1.AND.IFLAG(6).NE.1)GO TO 300
      XM2=(HDC(2)-HDP(2))/(YC-YP)
      XM4=(HDC(4)-HDP(4))/(YC-YP)
      IF(IFLAG(5).EQ.1)GO TO 68
      YINT=YP+(HDP(2)-HDP(4))/(XM4-XM2)
      GO TO 70
C   TO HERE IF REACHED EQUILIBRIUM DEPTH
   68 YINT=YP+(HSTAR-HDP(2))/XM2
      VUSED=VUSED+(HDP(4)-HDP(2))*(YINT-YP)
     1      +(XM4-XM2)*(YINT-YP)**2/2.0
      GO TO 400
   70 DVUSED=(HDP(4)-HDP(2))*(YINT-YP)+
     1       (XM4-XM2)*(YINT-YP)/2.0
      VUSED=VUSED+DVUSED
C     WRITE(6,26)IFLAG(5),IFLAG(6)
C     WRITE(6,22)IFLAG(5),IFLAG(6),YP,YC,HDP(2),HDP(4),HDC(2),
C    1 HDC(4)
C     WRITE(6,22)IA,IB,XM2,XM4,YINT,DVUSED,VUSED
      GO TO 400
  300 CALL DVOL(HDP,HDC,IA,IB,YP,YC,DVV)
C     IF(ITS.EQ.8)WRITE(8,*)ITS,I,IA,IB,YP,YC,DVV,HDP(2),
C    1  HDP(3),HDC(2),HDC(3)
C     IF(IFLAG(4).EQ.1)WRITE(6,22)IA,IB,VGEN,DVV,HDP(2),HDP(4),
C    1   HDC(2),HDC(4),YP
      VUSED=VUSED+DVV
      IF(IFLAG(1).GT.0)GO TO 400
      IA=1
      IB=3
      CALL DVOL(HDP,HDC,IA,IB,YP,YC,DVV)
C     IF(ITS.EQ.8)WRITE(8,*)ITS,I,IA,IB,YP,YC,DVV,VGEN,
C    1 HDP(1),HDP(3),HDC(1),HDC(3)
      VGEN=VGEN+DVV
  400 RETURN
      END
C
C     ----------------------------------------
      SUBROUTINE DVOL(HDP,HDC,IA,IB,YP,YC,DVV)
C     ----------------------------------------
      DIMENSION HDC(5),HDP(5)
   20 FORMAT(2I6,7F8.2)
      DVV=0.5*(YC-YP)*(HDP(IB)-HDP(IA)+HDC(IB)-HDC(IA))
C     WRITE(7,20)IA,IB,YP,YC,HDP(IA),HDP(IB),HDC(IA),HDC(IB),DVV
      RETURN
      END
C
C     ----------------------------------------------
      SUBROUTINE CROSS(HDP,HDC,IA,IB,YP,YC,YINT,HINT)
C     ----------------------------------------------
```

Figure A1. (Sheet 9 of 10)

```
      DIMENSION HDP(5),HDC(5)
   20 FORMAT(2I6,8F8.2)
C     WRITE(6,20)IA,IB,YP,YC,HDP(IA),HDP(IB),HDC(IA),HDC(IB)
      XMA=(HDC(IA)-HDP(IA))/(YC-YP)
      XMB=(HDC(IB)-HDP(IB))/(YC-YP)
      YINT=YP+(HDP(IA)-HDP(IB))/(XMB-XMA)
      HINT=HDP(IA)+XMA*(YINT-YP)
C     WRITE(6,20)IA,IB,XMA,XMB,YINT,HINT
      RETURN
      END
```

Figure A1. (Sheet 10 of 10)

```
EXAMPLE 1, CERC REPORT  OCTOBER 20,1991]  Run Descriptor
 ITYPE  IMAX   DY    AΦ     SΦ     SP     VADD    SEQ    HBERM   HSTAR   HMIX
0   780    1.0   0.10   0.10   0.10   140.0   0.05   -1.5    6.0     0.2
 JSED   LMAX   XMU   SIG
0    11     1.6   0.40        JSED, LMAX, & XMU,SIG  in phi units

 0.05    0.035    0.10   0.063    0.15    0.08    0.20   0.100     DI, AI
 0.30    0.120    0.40   0.140    0.50   0.160    0.70   0.170     (19 Pairs)
 0.80    0.200    1.00   0.210     2.0    0.27     5.0    0.36
 10.0     0.40    20.0    0.49    50.0    0.59   100.0    0.64     This Input
200.0     0.70   500.0    0.80  1000.0    0.86                     Same For
ITSMAX  DYEQ1                                                      All Runs
 40     20.0
```

Note: Blank Lines Above Have Been Added to Facilitate Annotation.

Figure A2. Listing of input file EQPR.INP for Example 1

EXAMPLE 1, CERC REPORT OCTOBER 20,1991

INPUT PAIRS OF DI,AI

.050	.035	.100	.063	.150	.080	.200	.100
.300	.120	.400	.140	.500	.160	.700	.170
.800	.200	1.000	.210	2.000	.270	5.000	.360
10.000	.400	20.000	.490	50.000	.590	100.000	.640
200.000	.700	500.000	.800	1000.000	.860		

INPUT PAIRS OF D(mm),P

.758	1.000	.642	.992	.543	.962	.460	.880
.390	.721	.330	.502	.279	.282	.237	.124
.200	.042	.170	.012	.144	.000		

I	Y(I)	HO(I)	HP(I)	HEQ(I)	AC(I)	VUSEDC(I)	PCC(I)
1	.000	-1.500	-1.500	.000	.000	.000	.000
2	1.000	-1.400	-1.500	.000	.000	.000	.000
3	2.000	-1.300	-1.500	.000	.000	.000	.000
4	3.000	-1.200	-1.500	.000	.000	.000	.000
5	4.000	-1.100	-1.500	.000	.000	.000	.000
6	5.000	-1.000	-1.500	.000	.000	.000	.000
7	6.000	-.900	-1.500	.000	.000	.000	.000
8	7.000	-.800	-1.500	.000	.000	.000	.000
9	8.000	-.700	-1.500	.000	.000	.000	.000
10	9.000	-.600	-1.500	.000	.000	.000	.000
11	10.000	-.500	-1.500	.000	.000	.000	.000
12	11.000	-.400	-1.500	.000	.000	.000	.000
13	12.000	-.300	-1.500	.000	.000	.000	.000
14	13.000	-.200	-1.500	.000	.000	.000	.000
15	14.000	-.100	-1.500	.000	.000	.000	.000
16	15.000	.000	-1.500	.000	.000	.000	.000
17	16.000	.057	-1.500	.000	.000	.000	.000
18	17.000	.100	-1.500	.000	.000	.000	.000
19	18.000	.138	-1.500	.000	.000	.000	.000
20	19.000	.173	-1.500	.000	.000	.000	.000
21	20.000	.205	-1.500	.000	.000	.000	.000
22	21.000	.236	-1.500	-1.500	.000	.000	.000
23	22.000	.266	-1.500	-1.414	.000	.000	.000
24	23.000	.295	-1.500	-1.364	.000	.000	.000
25	24.000	.322	-1.500	-1.314	.000	.000	.000
26	25.000	.349	-1.500	-1.264	.000	.000	.000
27	26.000	.375	-1.500	-1.214	.000	.000	.000
28	27.000	.400	-1.500	-1.164	.000	.000	.000
29	28.000	.425	-1.500	-1.114	.000	.000	.000
30	29.000	.449	-1.500	-1.064	.000	.000	.000
31	30.000	.473	-1.500	-1.014	.000	.000	.000
32	31.000	.496	-1.500	-.964	.000	.000	.000
33	32.000	.519	-1.500	-.914	.000	.000	.000
34	33.000	.541	-1.500	-.864	.000	.000	.000
35	34.000	.563	-1.500	-.814	.000	.000	.000
36	35.000	.585	-1.500	-.764	.000	.000	.000
37	36.000	.606	-1.500	-.714	.000	.000	.000
38	37.000	.628	-1.500	-.664	.000	.000	.000
39	38.000	.649	-1.500	-.614	.000	.000	.000

Figure A3. Listing of output file EQPR.OUT for Example 1 (Sheet 1 of 15)

40	39.000	.669	-1.500	-.564	.000	.000	.000
41	40.000	.689	-1.500	-.514	.000	.000	.000
42	41.000	.710	-1.500	-.464	.000	.000	.000
43	42.000	.730	-1.500	-.414	.000	.000	.000
44	43.000	.749	-1.500	-.364	.000	.000	.000
45	44.000	.769	-1.500	-.314	.000	.000	.000
46	45.000	.788	-1.500	-.264	.000	.000	.000
47	46.000	.807	-1.500	-.214	.000	.000	.000
48	47.000	.826	-1.500	-.164	.000	.000	.000
49	48.000	.845	-1.500	-.114	.000	.000	.000
50	49.000	.863	-1.500	-.064	.000	.000	.000
51	50.000	.882	-1.500	-.014	.156	6.346	.090
52	51.000	.900	-1.500	.032	.155	6.546	.093
53	52.000	.918	-1.500	.071	.155	6.746	.096
54	53.000	.936	-1.500	.107	.155	6.946	.099
55	54.000	.954	-1.500	.142	.154	7.146	.102
56	55.000	.972	-1.500	.176	.154	7.346	.104
57	56.000	.989	-1.500	.208	.154	7.546	.107
58	57.000	1.007	-1.500	.239	.153	7.746	.110
59	58.000	1.024	-1.500	.270	.153	7.946	.113
60	59.000	1.041	-1.429	.300	.153	8.146	.116
61	60.000	1.058	-1.329	.329	.152	8.346	.119
62	61.000	1.075	-1.229	.358	.152	8.546	.121
63	62.000	1.092	-1.129	.386	.152	8.746	.124
64	63.000	1.109	-1.029	.414	.151	8.946	.127
65	64.000	1.125	-.929	.441	.151	9.146	.130
66	65.000	1.142	-.829	.468	.151	9.346	.133
67	66.000	1.158	-.729	.495	.151	9.546	.136
68	67.000	1.175	-.629	.521	.150	9.746	.138
69	68.000	1.191	-.529	.547	.150	9.946	.141
70	69.000	1.207	-.429	.572	.150	10.146	.144
71	70.000	1.223	-.329	.597	.150	10.346	.147
72	71.000	1.239	-.229	.622	.149	10.546	.150
73	72.000	1.255	-.129	.647	.149	10.746	.153
74	73.000	1.271	-.029	.671	.149	10.946	.156
75	74.000	1.286	.071	.695	.149	11.146	.158
76	75.000	1.302	.171	.719	.148	11.346	.161
77	76.000	1.318	.271	.742	.148	11.546	.164
78	77.000	1.333	.371	.766	.148	11.746	.167
79	78.000	1.348	.471	.789	.148	11.946	.170
80	79.000	1.364	.571	.812	.147	12.146	.173
81	80.000	1.379	.671	.834	.147	12.346	.175
82	81.000	1.394	.771	.857	.147	12.546	.178
83	82.000	1.409	.871	.879	.147	12.746	.181
84	83.000	1.424	.971	.901	.146	13.198	.188
85	84.000	1.439	1.071	.923	.146	13.307	.189
86	85.000	1.454	1.171	.945	.146	13.493	.192
87	86.000	1.469	1.271	.967	.145	13.759	.195
88	87.000	1.484	1.371	.988	.145	14.102	.200
89	88.000	1.499	1.471	1.009	.144	14.525	.206
90	89.000	1.513	1.571	1.030	.144	14.829	.211
91	90.000	1.528	.000	1.051	.143	15.309	.217
92	91.000	1.542	.000	1.072	.143	15.782	.224
93	92.000	1.557	.000	1.092	.142	16.250	.231
94	93.000	1.571	.000	1.112	.142	16.712	.237
95	94.000	1.586	.000	1.132	.141	17.168	.244
96	95.000	1.600	.000	1.152	.140	17.619	.250
97	96.000	1.614	.000	1.172	.140	18.064	.257
98	97.000	1.628	.000	1.192	.139	18.503	.263
99	98.000	1.643	.000	1.211	.139	18.938	.269

Figure A3. (Sheet 2 of 15)

100	99.000	1.657	.000	1.230	.138	19.367	.275
101	100.000	1.671	.000	1.249	.138	19.791	.281
102	101.000	1.685	.000	1.268	.137	20.210	.287
103	102.000	1.699	.000	1.287	.137	20.625	.293
104	103.000	1.713	.000	1.305	.137	21.034	.299
105	104.000	1.726	.000	1.324	.137	21.[illegible]39	.305
106	105.000	1.740	.000	1.342	.136	21.839	.310
107	106.000	1.754	.000	1.361	.136	22.235	.316
108	107.000	1.768	.000	1.379	.136	22.626	.321
109	108.000	1.781	.000	1.397	.135	23.012	.327
110	109.000	1.795	.000	1.415	.135	23.395	.332
111	110.000	1.809	.000	1.433	.135	23.773	.338
112	111.000	1.822	.000	1.450	.134	24.147	.343
113	112.000	1.836	.000	1.468	.134	24.516	.348
114	113.000	1.849	.000	1.486	.134	24.882	.354
115	114.000	1.862	.000	1.503	.134	25.243	.359
116	115.000	1.876	.000	1.520	.133	25.601	.364
117	116.000	1.889	.000	1.537	.133	25.955	.369
118	117.000	1.902	.000	1.555	.133	26.304	.374
119	118.000	1.916	.000	1.572	.132	26.650	.379
120	119.000	1.929	.000	1.588	.132	26.992	.383
121	120.000	1.942	.000	1.605	.132	27.331	.388
122	121.000	1.955	.000	1.622	.132	27.666	.393
123	122.000	1.968	.000	1.639	.131	27.997	.398
124	123.000	1.981	.000	1.655	.131	28.325	.402
125	124.000	1.994	.000	1.672	.131	28.649	.407
126	125.000	2.007	.000	1.688	.131	28.970	.412
127	126.000	2.020	.000	1.704	.130	29.287	.416
128	127.000	2.033	.000	1.721	.130	29.602	.421
129	128.000	2.046	.000	1.737	.130	29.912	.425
130	129.000	2.059	.000	1.753	.130	30.220	.429
131	130.000	2.072	.000	1.769	.129	30.524	.434
132	131.000	2.084	.000	1.785	.129	30.826	.438
133	132.000	2.097	.000	1.800	.129	31.124	.442
134	133.000	2.110	.000	1.816	.129	31.419	.446
135	134.000	2.122	.000	1.832	.129	31.711	.451
136	135.000	2.135	.000	1.847	.128	32.000	.455
137	136.000	2.148	.000	1.863	.128	32.286	.459
138	137.000	2.160	.000	1.878	.128	32.569	.463
139	138.000	2.173	.000	1.894	.128	32.849	.467
140	139.000	2.185	.000	1.909	.127	33.127	.471
141	140.000	2.198	.000	1.924	.127	33.401	.475
142	141.000	2.210	.000	1.940	.127	33.673	.478
143	142.000	2.222	.000	1.955	.127	33.942	.482
144	143.000	2.235	.000	1.970	.127	34.209	.486
145	144.000	2.247	.000	1.985	.126	34.473	.490
146	145.000	2.260	.000	2.000	.126	34.734	.493
147	146.000	2.272	.000	2.014	.126	34.993	.497
148	147.000	2.284	.000	2.029	.126	35.249	.501
149	148.000	2.296	.000	2.044	.126	35.502	.504
150	149.000	2.308	.000	2.059	.126	35.754	.508
151	150.000	2.321	.000	2.073	.125	36.002	.511
152	151.000	2.333	.000	2.088	.125	36.249	.515
153	152.000	2.345	.000	2.102	.125	36.492	.518
154	153.000	2.357	.000	2.117	.125	36.734	.522
155	154.000	2.369	.000	2.131	.125	36.973	.525
156	155.000	2.381	.000	2.145	.125	37.210	.529
157	156.000	2.393	.000	2.160	.124	37.445	.532
158	157.000	2.405	.000	2.174	.124	37.677	.535
159	158.000	2.417	.000	2.188	.124	37.907	.539

Figure A3. (Sheet 3 of 15)

160	159.000	2.429	.000	2.202	.124	38.135	.542
161	160.000	2.441	.000	2.216	.124	38.361	.545
162	161.000	2.453	.000	2.230	.124	38.585	.548
163	162.000	2.465	.000	2.244	.124	38.806	.551
164	163.000	2.476	.000	2.258	.123	39.025	.554
165	164.000	2.488	.000	2.272	.123	39.243	.558
166	165.000	2.500	.000	2.286	.123	39.458	.561
167	166.000	2.512	.000	2.300	.123	39.671	.564
168	167.000	2.523	.000	2.313	.123	39.882	.567
169	168.000	2.535	.000	2.327	.123	40.091	.570
170	169.000	2.547	.000	2.341	.123	40.299	.573
171	170.000	2.559	.000	2.354	.122	40.504	.575
172	171.000	2.570	.000	2.368	.122	40.707	.578
173	172.000	2.582	.000	2.381	.122	40.909	.581
174	173.000	2.593	.000	2.395	.122	41.108	.584
175	174.000	2.605	.000	2.408	.122	41.306	.587
176	175.000	2.616	.000	2.422	.122	41.502	.590
177	176.000	2.628	.000	2.435	.122	41.696	.592
178	177.000	2.639	.000	2.448	.122	41.888	.595
179	178.000	2.651	.000	2.461	.121	42.078	.598
180	179.000	2.662	.000	2.475	.121	42.267	.600
181	180.000	2.674	.000	2.488	.121	42.454	.603
182	181.000	2.685	.000	2.501	.121	42.639	.606
183	182.000	2.697	.000	2.514	.121	42.823	.608
184	183.000	2.708	.000	2.527	.121	43.005	.611
185	184.000	2.719	.000	2.540	.121	43.185	.614
186	185.000	2.731	.000	2.553	.121	43.363	.616
187	186.000	2.742	.000	2.566	.120	43.540	.619
188	187.000	2.753	.000	2.579	.120	43.715	.621
189	188.000	2.765	.000	2.592	.120	43.889	.624
190	189.000	2.776	.000	2.605	.120	44.061	.626
191	190.000	2.787	.000	2.617	.120	44.231	.628
192	191.000	2.798	.000	2.630	.120	44.400	.631
193	192.000	2.809	.000	2.643	.120	44.568	.633
194	193.000	2.821	.000	2.655	.120	44.734	.636
195	194.000	2.832	.000	2.668	.120	44.898	.638
196	195.000	2.843	.000	2.681	.119	45.061	.640
197	196.000	2.854	.000	2.693	.119	45.222	.642
198	197.000	2.865	.000	2.706	.119	45.382	.645
199	198.000	2.876	.000	2.718	.119	45.541	.647
200	199.000	2.887	.000	2.731	.119	45.698	.649
201	200.000	2.898	.000	2.743	.119	45.853	.651
202	201.000	2.909	.000	2.756	.119	46.008	.654
203	202.000	2.920	.000	2.768	.119	46.161	.656
204	203.000	2.931	.000	2.780	.119	46.312	.658
205	204.000	2.942	.000	2.793	.119	46.462	.660
206	205.000	2.953	.000	2.805	.118	46.611	.662
207	206.000	2.964	.000	2.817	.118	46.759	.664
208	207.000	2.975	.000	2.829	.118	46.905	.666
209	208.000	2.986	.000	2.841	.118	47.050	.668
210	209.000	2.997	.000	2.854	.118	47.193	.670
211	210.000	3.007	.000	2.866	.118	47.336	.673
212	211.000	3.018	.000	2.878	.118	47.477	.675
213	212.000	3.029	.000	2.890	.118	47.617	.676
214	213.000	3.040	.000	2.902	.118	47.755	.678
215	214.000	3.051	.000	2.914	.118	47.893	.680
216	215.000	3.061	.000	2.926	.117	48.029	.682
217	216.000	3.072	.000	2.938	.117	48.164	.684
218	217.000	3.083	.000	2.950	.117	48.297	.686
219	218.000	3.094	.000	2.962	.117	48.430	.688

Figure A3. (Sheet 4 of 15)

220	219.000	3.104	.000	2.973	.117	48.561	.690
221	220.000	3.115	.000	2.985	.117	48.692	.692
222	221.000	3.126	.000	2.997	.117	48.821	.694
223	222.000	3.136	.000	3.009	.117	48.949	.695
224	223.000	3.147	.000	3.021	.117	49.076	.697
225	224.000	3.157	.000	3.032	.117	49.202	.699
226	225.000	3.168	.000	3.044	.117	49.326	.701
227	226.000	3.179	.000	3.056	.117	49.450	.703
228	227.000	3.189	.000	3.067	.116	49.572	.704
229	228.000	3.200	.000	3.079	.116	49.694	.706
230	229.000	3.210	.000	3.090	.116	49.814	.708
231	230.000	3.221	.000	3.102	.116	49.934	.709
232	231 000	3.231	.000	3.114	.116	50.052	.711
233	232.000	3.242	.000	3.125	.116	50.169	.713
234	233.000	3.252	.000	3.137	.116	50.286	.714
235	234.000	3.263	.000	3.148	.116	50.401	.716
236	235.000	3.273	.000	3.159	.116	50.515	.718
237	236.000	3.284	.000	3.171	.116	50.628	.719
238	237.000	3.294	.000	3.182	.116	50.741	.721
239	238.000	3.304	.000	3.194	.116	50.852	.722
240	239.000	3.315	.000	3.205	.116	50.963	.724
241	240.000	3.325	.000	3.216	.115	51.072	.726
242	241.000	3.336	.000	3.227	.115	51.181	.727
243	242.000	3.346	.000	3.239	.115	51.288	.729
244	243.000	3.356	.000	3.250	.115	51.395	.730
245	244.000	3.367	.000	3.261	.115	51.501	.732
246	245.000	3.377	.000	3.272	.115	51.606	.733
247	246.000	3.387	.000	3.284	.115	51.710	.735
248	247.000	3.397	.000	3.295	.115	51.813	.736
249	248.000	3.408	.000	3.306	.115	51.915	.738
250	249.000	3.418	.000	3.317	.115	52.017	.739
251	250.000	3.428	.000	3.328	.115	52.118	.740
252	251.000	3.438	.000	3.339	.115	52.217	.742
253	252.000	3.449	.000	3.350	.114	52.316	.743
254	253.000	3.459	.000	3.361	.114	52.415	.745
255	254.000	3.469	.000	3.372	.114	52.512	.746
256	255.000	3.479	.000	3.383	.114	52.608	.747
257	256.000	3.489	.000	3.394	.114	52.704	.749
258	257.000	3.499	.000	3.405	.114	52.799	.750
259	258.000	3.509	.000	3.416	.114	52.893	.751
260	259.000	3.520	.000	3.427	.114	52.987	.753
261	260.000	3.530	.000	3.437	.114	53.080	.754
262	261.000	3.540	.000	3.448	.114	53.172	.755
263	262.000	3.550	.000	3.459	.114	53.263	.757
264	263.000	3.560	.000	3.470	.114	53.353	.758
265	264.000	3.570	.000	3.481	.114	53.443	.759
266	265.000	3.580	.000	3.491	.114	53.532	.761
267	266.000	3.590	.000	3.502	.113	53.621	.762
268	267.000	3.600	.000	3.513	.113	53.708	.763
269	268.000	3.610	.000	3.523	.113	53.795	.764
270	269.000	3.620	.000	3.534	.113	53.882	.766
271	270.000	3.630	.000	3.545	.113	53.967	.767
272	271.000	3.640	.000	3.555	.113	54.052	.768
273	272.000	3.650	.000	3.566	.113	54.137	.769
274	273.000	3.660	.000	3.576	.113	54.220	.770
275	274.000	3.670	.000	3.587	.113	54.303	.771
276	275.000	3.680	.000	3.598	.113	54.386	.773
277	276.000	3.690	.000	3.608	.113	54.467	.774
278	277.000	3.699	.000	3.619	.113	54.549	.775
279	278.000	3.709	.000	3.629	.113	54.629	.776

Figure A3. (Sheet 5 of 15)

280	279.000	3.719	.000	3.639	.113	54.709	.777
281	280.000	3.729	.000	3.650	.113	54.788	.778
282	281.000	3.739	.000	3.660	.113	54.867	.780
283	282.000	3.749	.000	3.671	.112	54.945	.781
284	283.000	3.758	.000	3.681	.112	55.023	.782
285	284.000	3.768	.000	3.692	.112	55.100	.783
286	285.000	3.778	.000	3.702	.112	55.176	.784
287	286.000	3.788	.000	3.712	.112	55.252	.785
288	287.000	3.798	.000	3.723	.112	55.328	.786
289	288.000	3.807	.000	3.733	.112	55.402	.787
290	289.000	3.817	.000	3.743	.112	55.477	.788
291	290.000	3.827	.000	3.753	.112	55.550	.789
292	291.000	3.837	.000	3.764	.112	55.623	.790
293	292.000	3.846	.000	3.774	.112	55.696	.791
294	293.000	3.856	.000	3.784	.112	55.768	.792
295	294.000	3.866	.000	3.794	.112	55.840	.793
296	295.000	3.875	.000	3.804	.112	55.911	.794
297	296.000	3.885	.000	3.815	.112	55.982	.795
298	297.000	3.895	.000	3.825	.112	56.052	.796
299	298.000	3.904	.000	3.835	.112	56.121	.797
300	299.000	3.914	.000	3.845	.112	56.190	.798
301	300.000	3.924	.000	3.855	.111	56.259	.799
302	301.000	3.933	.000	3.865	.111	56.327	.800
303	302.000	3.943	.000	3.875	.111	56.395	.801
304	303.000	3.952	.000	3.885	.111	56.462	.802
305	304.000	3.962	.000	3.895	.111	56.529	.803
306	305.000	3.972	.000	3.905	.111	56.596	.804
307	306.000	3.981	.000	3.915	.111	56.662	.805
308	307.000	3.991	.000	3.925	.111	56.727	.806
309	308.000	4.000	.000	3.935	.111	56.792	.807
310	309.000	4.010	.000	3.945	.111	56.857	.808
311	310.000	4.019	.000	3.955	.111	56.921	.809
312	311.000	4.029	.000	3.965	.111	56.985	.810
313	312.000	4.038	.000	3.975	.111	57.048	.810
314	313.000	4.048	.000	3.985	.111	57.111	.811
315	314.000	4.057	.000	3.995	.111	57.174	.812
316	315.000	4.067	.000	4.005	.111	57.236	.813
317	316.000	4.076	.000	4.015	.111	57.298	.814
318	317.000	4.086	.000	4.024	.111	57.359	.815
319	318.000	4.095	.000	4.034	.111	57.421	.816
320	319.000	4.104	.000	4.044	.111	57.481	.817
321	320.000	4.114	.000	4.054	.110	57.541	.818
322	321.000	4.123	.000	4.064	.110	57.601	.818
323	322.000	4.133	.000	4.073	.110	57.661	.819
324	323.000	4.142	.000	4.083	.110	57.720	.820
325	324.000	4.151	.000	4.093	.110	57.779	.821
326	325.000	4.161	.000	4.102	.110	57.838	.822
327	326.000	4.170	.000	4.112	.110	57.896	.823
328	327.000	4.180	.000	4.122	.110	57.954	.823
329	328.000	4.189	.000	4.132	.110	58.011	.824
330	329.000	4.198	.000	4.141	.110	58.068	.825
331	330.000	4.208	.000	4.151	.110	58.125	.826
332	331.000	4.217	.000	4.160	.110	58.182	.827
333	332.000	4.226	.000	4.170	.110	58.238	.827
334	333.000	4.235	.000	4.180	.110	58.294	.828
335	334.000	4.245	.000	4.189	.110	58.349	.829
336	335.000	4.254	.000	4.199	.110	58.405	.830
337	336.000	4.263	.000	4.208	.110	58.460	.831
338	337.000	4.273	.000	4.218	.110	58.514	.831
339	338.000	4.282	.000	4.228	.110	58.569	.832

Figure A3. (Sheet 6 of 15)

340	339.000	4.291	.000	4.237	.110	58.623	.833
341	340.000	4.300	.000	4.247	.110	58.677	.834
342	341.000	4.310	.000	4.256	.110	58.730	.834
343	342.000	4.319	.000	4.266	.110	58.783	.835
344	343.000	4.328	.000	4.275	.109	58.836	.836
345	344.000	4.337	.000	4.285	.109	58.889	.837
346	345.000	4.346	.000	4.294	.109	58.941	.837
347	346.000	4.355	.000	4.303	.109	58.994	.838
348	347.000	4.365	.000	4.313	.109	59.046	.839
349	348.000	4.374	.000	4.322	.109	59.097	.840
350	349.000	4.383	.000	4.332	.109	59.149	.840
351	350.000	4.392	.000	4.341	.109	59.200	.841
352	351.000	4.401	.000	4.350	.109	59.251	.842
353	352.000	4.410	.000	4.360	.109	59.302	.843
354	353.000	4.419	.000	4.369	.109	59.352	.843
355	354.000	4.429	.000	4.378	.109	59.402	.844
356	355.000	4.438	.000	4.388	.109	59.452	.845
357	356.000	4.447	.000	4.397	.109	59.502	.845
358	357.000	4.456	.000	4.406	.109	59.552	.846
359	358.000	4.465	.000	4.416	.109	59.601	.847
360	359.000	4.474	.000	4.425	.109	59.650	.847
361	360.000	4.483	.000	4.434	.109	59.699	.848
362	361.000	4.492	.000	4.444	.109	59.748	.849
363	362.000	4.501	.000	4.453	.109	59.796	.850
364	363.000	4.510	.000	4.462	.109	59.845	.850
365	364.000	4.519	.000	4.471	.109	59.893	.851
366	365.000	4.528	.000	4.480	.109	59.941	.852
367	366.000	4.537	.000	4.490	.109	59.988	.852
368	367.000	4.546	.000	4.499	.109	60.036	.853
369	368.000	4.555	.000	4.508	.109	60.083	.854
370	369.000	4.564	.000	4.517	.108	60.131	.854
371	370.000	4.573	.000	4.526	.108	60.178	.855
372	371.000	4.582	.000	4.535	.108	60.224	.856
373	372.000	4.591	.000	4.545	.108	60.271	.856
374	373.000	4.600	.000	4.554	.108	60.318	.857
375	374.000	4.609	.000	4.563	.108	60.364	.858
376	375.000	4.618	.000	4.572	.108	60.410	.858
377	376.000	4.627	.000	4.581	.108	60.456	.859
378	377.000	4.636	.000	4.590	.108	60.502	.860
379	378.000	4.645	.000	4.599	.108	60.548	.860
380	379.000	4.654	.000	4.608	.108	60.593	.861
381	380.000	4.663	.000	4.617	.108	60.639	.862
382	381.000	4.671	.000	4.626	.108	60.684	.862
383	382.000	4.680	.000	4.635	.108	60.729	.863
384	383.000	4.689	.000	4.644	.108	60.774	.863
385	384.000	4.698	.000	4.653	.108	60.819	.864
386	385.000	4.707	.000	4.662	.108	60.864	.865
387	386.000	4.716	.000	4.671	.108	60.909	.865
388	387.000	4.725	.000	4.680	.108	60.953	.866
389	388.000	4.733	.000	4.689	.108	60.998	.867
390	389.000	4.742	.000	4.698	.108	61.042	.867
391	390.000	4.751	.000	4.707	.108	61.086	.868
392	391.000	4.760	.000	4.716	.108	61.130	.868
393	392.000	4.769	.000	4.725	.108	61.174	.869
394	393.000	4.778	.000	4.734	.108	61.218	.870
395	394.000	4.786	.000	4.743	.108	61.262	.870
396	395.000	4.795	.000	4.752	.108	61.305	.871
397	396.000	4.804	.000	4.760	.108	61.349	.872
398	397.000	4.813	.000	4.769	.108	61.393	.872
399	398.000	4.821	.000	4.778	.107	61.436	.873

Figure A3. (Sheet 7 of 15)

400	399.000	4.830	.000	4.787	.107	61.479	.873
401	400.000	4.839	.000	4.796	.107	61.522	.874
402	401.000	4.848	.000	4.805	.107	61.566	.875
403	402.000	4.856	.000	4.813	.107	61.609	.875
404	403.000	4.865	.000	4.822	.107	61.652	.876
405	404.000	4.874	.000	4.831	.107	61.694	.877
406	405.000	4.883	.000	4.840	.107	61.737	.877
407	406.000	4.891	.000	4.849	.107	61.780	.878
408	407.000	4.900	.000	4.857	.107	61.823	.878
409	408.000	4.909	.000	4.866	.107	61.865	.879
410	409.000	4.917	.000	4.875	.107	61.908	.880
411	410.000	4.926	.000	4.884	.107	61.951	.880
412	411.000	4.935	.000	4.892	.107	61.993	.881
413	412.000	4.943	.000	4.901	.107	62.036	.881
414	413.000	4.952	.000	4.910	.107	62.078	.882
415	414.000	4.961	.000	4.918	.107	62.121	.883
416	415.000	4.969	.000	4.927	.107	62.163	.883
417	416.000	4.978	.000	4.936	.107	62.205	.884
418	417.000	4.987	.000	4.944	.107	62.248	.884
419	418.000	4.995	.000	4.953	.107	62.290	.885
420	419.000	5.004	.000	4.961	.106	62.332	.886
421	420.000	5.012	.000	4.970	.106	62.375	.886
422	421.000	5.021	.000	4.979	.106	62.417	.887
423	422.000	5.030	.000	4.987	.106	62.460	.887
424	423.000	5.038	.000	4.996	.106	62.502	.888
425	424.000	5.047	.000	5.004	.106	62.544	.889
426	425.000	5.055	.000	5.013	.106	62.587	.889
427	426.000	5.064	.000	5.021	.106	62.629	.890
428	427.000	5.073	.000	5.030	.106	62.672	.890
429	428.000	5.081	.000	5.038	.106	62.715	.891
430	429.000	5.090	.000	5.047	.106	62.757	.892
431	430.000	5.098	.000	5.055	.106	62.800	.892
432	431.000	5.107	.000	5.064	.106	62.843	.893
433	432.000	5.115	.000	5.072	.106	62.885	.893
434	433.000	5.124	.000	5.081	.106	62.928	.894
435	434.000	5.132	.000	5.089	.106	62.971	.895
436	435.000	5.141	.000	5.098	.106	63.014	.895
437	436.000	5.149	.000	5.106	.106	63.057	.896
438	437.000	5.158	.000	5.115	.105	63.100	.896
439	438.000	5.166	.000	5.123	.105	63.144	.897
440	439.000	5.175	.000	5.131	.105	63.187	.898
441	440.000	5.183	.000	5.140	.105	63.230	.898
442	441.000	5.192	.000	5.148	.105	63.274	.899
443	442.000	5.200	.000	5.156	.105	63.318	.900
444	443.000	5.209	.000	5.165	.105	63.361	.900
445	444.000	5.217	.000	5.173	.105	63.405	.901
446	445.000	5.226	.000	5.181	.105	63.449	.901
447	446.000	5.234	.000	5.190	.105	63.493	.902
448	447.000	5.242	.000	5.198	.105	63.538	.903
449	448.000	5.251	.000	5.206	.105	63.582	.903
450	449.000	5.259	.000	5.215	.105	63.627	.904
451	450.000	5.268	.000	5.223	.105	63.671	.905
452	451.000	5.276	.000	5.231	.105	63.716	.905
453	452.000	5.285	.000	5.239	.105	63.761	.906
454	453.000	5.293	.000	5.248	.105	63.806	.907
455	454.000	5.301	.000	5.256	.105	63.852	.907
456	455.000	5.310	.000	5.264	.104	63.897	.908
457	456.000	5.318	.000	5.272	.104	63.943	.908
458	457.000	5.326	.000	5.280	.104	63.989	.909
459	458.000	5.335	.000	5.289	.104	64.035	.910

Figure A3. (Sheet 8 of 15)

460	459.000	5.343	.000	5.297	.104	64.081	.910
461	460.000	5.352	.000	5.305	.104	64.127	.911
462	461.000	5.360	.000	5.313	.104	64.174	.912
463	462.000	5.368	.000	5.321	.104	64.221	.912
464	463.000	5.377	.000	5.329	.104	64.268	.913
465	464.000	5.385	.000	5.338	.104	64.315	.914
466	465.000	5.393	.000	5.346	.104	64.362	.914
467	466.000	5.402	.000	5.354	.104	64.410	.915
468	467.000	5.410	.000	5.362	.104	64.458	.916
469	468.000	5.418	.000	5.370	.104	64.506	.916
470	469.000	5.426	.000	5.378	.104	64.555	.917
471	470.000	5.435	.000	5.386	.104	64.603	.918
472	471.000	5.443	.000	5.394	.104	64.652	.919
473	472.000	5.451	.000	5.402	.103	64.701	.919
474	473.000	5.460	.000	5.410	.103	64.751	.920
475	474.000	5.468	.000	5.418	.103	64.800	.921
476	475.000	5.476	.000	5.426	.103	64.850	.921
477	476.000	5.484	.000	5.434	.103	64.900	.922
478	477.000	5.493	.000	5.442	.103	64.951	.923
479	478.000	5.501	.000	5.450	.103	65.002	.923
480	479.000	5.509	.000	5.458	.103	65.053	.924
481	480.000	5.517	.000	5.466	.103	65.104	.925
482	481.000	5.526	.000	5.474	.103	65.156	.926
483	482.000	5.534	.000	5.482	.103	65.208	.926
484	483.000	5.542	.000	5.490	.103	65.260	.927
485	484.000	5.550	.000	5.497	.103	65.313	.928
486	485.000	5.558	.000	5.505	.103	65.366	.929
487	486.000	5.567	.000	5.513	.103	65.419	.929
488	487.000	5.575	.000	5.521	.103	65.473	.930
489	488.000	5.583	.000	5.529	.102	65.527	.931
490	489.000	5.591	.000	5.537	.102	65.581	.932
491	490.000	5.599	.000	5.545	.102	65.636	.932
492	491.000	5.608	.000	5.552	.102	65.691	.933
493	492.000	5.616	.000	5.560	.102	65.746	.934
494	493.000	5.624	.000	5.568	.102	65.802	.935
495	494.000	5.632	.000	5.576	.102	65.858	.936
496	495.000	5.640	.000	5.584	.102	65.914	.936
497	496.000	5.648	.000	5.591	.102	65.971	.937
498	497.000	5.657	.000	5.599	.102	66.029	.938
499	498.000	5.665	.000	5.607	.102	66.086	.939
500	499.000	5.673	.000	5.615	.102	66.145	.940
501	500.000	5.681	.000	5.622	.102	66.203	.941
502	501.000	5.689	.000	5.630	.102	66.262	.941
503	502.000	5.697	.000	5.638	.101	66.322	.942
504	503.000	5.705	.000	5.645	.101	66.381	.943
505	504.000	5.713	.000	5.653	.101	66.442	.944
506	505.000	5.722	.000	5.661	.101	66.502	.945
507	506.000	5.730	.000	5.668	.101	66.563	.946
508	507.000	5.738	.000	5.676	.101	66.625	.947
509	508.000	5.746	.000	5.684	.101	66.687	.947
510	509.000	5.754	.000	5.691	.101	66.750	.948
511	510.000	5.762	.000	5.699	.101	66.813	.949
512	511.000	5.770	.000	5.706	.101	66.876	.950
513	512.000	5.778	.000	5.714	.101	66.940	.951
514	513.000	5.786	.000	5.721	.101	67.005	.952
515	514.000	5.794	.000	5.729	.101	67.070	.953
516	515.000	5.802	.000	5.737	.100	67.135	.954
517	516.000	5.810	.000	5.744	.100	67.201	.955
518	517.000	5.818	.000	5.752	.100	67.268	.956
519	518.000	5.826	.000	5.759	.100	67.335	.957

Figure A3. (Sheet 9 of 15)

520	519.000	5.834	.000	5.767	.100	67.403	.958
521	520.000	5.842	.000	5.774	.100	67.471	.959
522	521.000	5.850	.000	5.781	.099	67.540	.960
523	522.000	5.859	.000	5.789	.099	67.609	.961
524	523.000	5.867	.000	5.796	.099	67.679	.962
525	524.000	5.875	.000	5.804	.098	67.750	.963
526	525.000	5.883	.000	5.811	.098	67.821	.964
527	526.000	5.891	.000	5.818	.097	67.893	.965
528	527.000	5.899	.000	5.825	.097	67.966	.966
529	528.000	5.906	.000	5.832	.097	68.040	.967
530	529.000	5.914	.000	5.839	.096	68.114	.968
531	530.000	5.922	.000	5.847	.096	68.190	.969
532	531.000	5.930	.000	5.854	.095	68.266	.970
533	532.000	5.938	.000	5.861	.095	68.343	.971
534	533.000	5.946	.000	5.867	.094	68.422	.972
535	534.000	5.954	.000	5.874	.094	68.501	.97[illegible]
536	535.000	5.962	.000	5.881	.093	68.582	.974
537	536.000	5.970	.000	5.888	.093	68.663	.976
538	537.000	5.978	.000	5.895	.093	68.746	.977
539	538.000	5.986	.000	5.901	.092	68.830	.978
540	539.000	5.994	.000	5.908	.092	68.916	.979
541	540.000	6.002	.000	5.915	.091	69.002	.980
542	541.000	6.010	.000	5.921	.091	69.090	.982
543	542.000	6.018	.000	5.928	.090	69.180	.983
544	543.000	6.026	.000	5.934	.090	69.270	.984
545	544.000	6.034	.000	5.941	.089	69.363	.985
546	545.000	6.041	.000	5.947	.088	69.457	.987
547	546.000	6.049	.000	5.953	.088	69.552	.988
548	547.000	6.057	.000	5.959	.087	69.649	.990
549	548.000	6.065	.000	5.965	.086	69.748	.991
550	549.000	6.073	.000	5.972	.084	69.848	.992
551	550.000	6.081	.000	5.977	.083	69.951	.994
552	551.000	6.089	.000	5.983	.082	70.056	.995
553	552.000	6.097	.000	5.989	.081	70.162	.997
554	553.000	6.105	.000	5.994	.079	70.271	.998
555	554.000	6.112	.000	6.000	.078	70.383	1.000
556	555.000	6.120	.000	6.005	.078	70.387	1.000
557	556.000	6.128	.000	6.010	.000	.000	.000
558	557.000	6.136	.000	.000	.000	.000	.000
559	558.000	6.144	.000	.000	.000	.000	.000
560	559.000	6.152	.000	.000	.000	.000	.000
561	560.000	6.159	.000	.000	.000	.000	.000
562	561.000	6.167	.000	.000	.000	.000	.000
563	562.000	6.175	.000	.000	.000	.000	.000
564	563.000	6.183	.000	.000	.000	.000	.000
565	564.000	6.191	.000	.000	.000	.000	.000
566	565.000	6.199	.000	.000	.000	.000	.000
567	566.000	6.206	.000	.000	.000	.000	.000
568	567.000	6.214	.000	.000	.000	.000	.000
569	568.000	6.222	.000	.000	.000	.000	.000
570	569.000	6.230	.000	.000	.000	.000	.000
571	570.000	6.238	.000	.000	.000	.000	.000
572	571.000	6.245	.000	.000	.000	.000	.000
573	572.000	6.253	.000	.000	.000	.000	.000
574	573.000	6.261	.000	.000	.000	.000	.000
575	574.000	6.269	.000	.000	.000	.000	.000
576	575.000	6.276	.000	.000	.000	.000	.000
577	576.000	6.284	.000	.000	.000	.000	.000
578	577.000	6.292	.000	.000	.000	.000	.000
579	578.000	6.300	.000	.000	.000	.000	.000

Figure A3. (Sheet 10 of 15)

580	579.000	6.307	.000	.000	.000	.000	.000
581	580.000	6.315	.000	.000	.000	.000	.000
582	581.000	6.323	.000	.000	.000	.000	.000
583	582.000	6.331	.000	.000	.000	.000	.000
584	583.000	6.338	.000	.000	.000	.000	.000
585	584.000	6.346	.000	.000	.000	.000	.000
586	585.000	6.354	.000	.000	.000	.000	.000
587	586.000	6.361	.000	.000	.000	.000	.000
588	587.000	6.369	.000	.000	.000	.000	.000
589	588.000	6.377	.000	.000	.000	.000	.000
590	589.000	6.385	.000	.000	.000	.000	.000
591	590.000	6.392	.000	.000	.000	.000	.000
592	591.000	6.400	.000	.000	.000	.000	.000
593	592.000	6.408	.000	.000	.000	.000	.000
594	593.000	6.415	.000	.000	.000	.000	.000
595	594.000	6.423	.000	.000	.000	.000	.000
596	595.000	6.431	.000	.000	.000	.000	.000
597	596.000	6.438	.000	.000	.000	.000	.000
598	597.000	6.446	.000	.000	.000	.000	.000
599	598.000	6.454	.000	.000	.000	.000	.000
600	599.000	6.461	.000	.000	.000	.000	.000
601	600.000	6.469	.000	.000	.000	.000	.000
602	601.000	6.477	.000	.000	.000	.000	.000
603	602.000	6.484	.000	.000	.000	.000	.000
604	603.000	6.492	.000	.000	.000	.000	.000
605	604.000	6.500	.000	.000	.000	.000	.000
606	605.000	6.507	.000	.000	.000	.000	.000
607	606.000	6.515	.000	.000	.000	.000	.000
608	607.000	6.523	.000	.000	.000	.000	.000
609	608.000	6.530	.000	.000	.000	.000	.000
610	609.000	6.538	.000	.000	.000	.000	.000
611	610.000	6.545	.000	.000	.000	.000	.000
612	611.000	6.553	.000	.000	.000	.000	.000
613	612.000	6.561	.000	.000	.000	.000	.000
614	613.000	6.568	.000	.000	.000	.000	.000
615	614.000	6.576	.000	.000	.000	.000	.000
616	615.000	6.583	.000	.000	.000	.000	.000
617	616.000	6.591	.000	.000	.000	.000	.000
618	617.000	6.599	.000	.000	.000	.000	.000
619	618.000	6.606	.000	.000	.000	.000	.000
620	619.000	6.614	.000	.000	.000	.000	.000
621	620.000	6.621	.000	.000	.000	.000	.000
622	621.000	6.629	.000	.000	.000	.000	.000
623	622.000	6.636	.000	.000	.000	.000	.000
624	623.000	6.644	.000	.000	.000	.000	.000
625	624.000	6.652	.000	.000	.000	.000	.000
626	625.000	6.659	.000	.000	.000	.000	.000
627	626.000	6.667	.000	.000	.000	.000	.000
628	627.000	6.674	.000	.000	.000	.000	.000
629	628.000	6.682	.000	.000	.000	.000	.000
630	629.000	6.689	.000	.000	.000	.000	.000
631	630.000	6.697	.000	.000	.000	.000	.000
632	631.000	6.704	.000	.000	.000	.000	.000
633	632.000	6.712	.000	.000	.000	.000	.000
634	633.000	6.719	.000	.000	.000	.000	.000
635	634.000	6.727	.000	.000	.000	.000	.000
636	635.000	6.734	.000	.000	.000	.000	.000
637	636.000	6.742	.000	.000	.000	.000	.000
638	637.000	6.749	.000	.000	.000	.000	.000
639	638.000	6.757	.000	.000	.000	.000	.000

Figure A3. (Sheet 11 of 15)

640	639.000	6.764	.000	.000	.000	.000	.000
641	640.000	6.772	.000	.000	.000	.000	.000
642	641.000	6.779	.000	.000	.000	.000	.000
643	642.000	6.787	.000	.000	.000	.000	.000
644	643.000	6.794	.000	.000	.000	.000	.000
645	644.000	6.802	.000	.000	.000	.000	.000
646	645.000	6.809	.000	.000	.000	.000	.000
647	646.000	6.817	.000	.000	.000	.000	.000
648	647.000	6.824	.000	.000	.000	.000	.000
649	648.000	6.832	.000	.000	.000	.000	.000
650	649.000	6.839	.000	.000	.000	.000	.000
651	650.000	6.847	.000	.000	.000	.000	.000
652	651.000	6.854	.000	.000	.000	.000	.000
653	652.000	6.862	.000	.000	.000	.000	.000
654	653.000	6.869	.000	.000	.000	.000	.000
655	654.000	6.877	.000	.000	.000	.000	.000
656	655.000	6.884	.000	.000	.000	.000	.000
657	656.000	6.891	.000	.000	.000	.000	.000
658	657.000	6.899	.000	.000	.000	.000	.000
659	658.000	6.906	.000	.000	.000	.000	.000
660	659.000	6.914	.000	.000	.000	.000	.000
661	660.000	6.921	.000	.000	.000	.000	.000
662	661.000	6.929	.000	.000	.000	.000	.000
663	662.000	6.936	.000	.000	.000	.000	.000
664	663.000	6.943	.000	.000	.000	.000	.000
665	664.000	6.951	.000	.000	.000	.000	.000
666	665.000	6.958	.000	.000	.000	.000	.000
667	666.000	6.966	.000	.000	.000	.000	.000
668	667.000	6.973	.000	.000	.000	.000	.000
669	668.000	6.980	.000	.000	.000	.000	.000
670	669.000	6.988	.000	.000	.000	.000	.000
671	670.000	6.995	.000	.000	.000	.000	.000
672	671.000	7.002	.000	.000	.000	.000	.000
673	672.000	7.010	.000	.000	.000	.000	.000
674	673.000	7.017	.000	.000	.000	.000	.000
675	674.000	7.025	.000	.000	.000	.000	.000
676	675.000	7.032	.000	.000	.000	.000	.000
677	676.000	7.039	.000	.000	.000	.000	.000
678	677.000	7.047	.000	.000	.000	.000	.000
679	678.000	7.054	.000	.000	.000	.000	.000
680	679.000	7.061	.000	.000	.000	.000	.000
681	680.000	7.069	.000	.000	.000	.000	.000
682	681.000	7.076	.000	.000	.000	.000	.000
683	682.000	7.083	.000	.000	.000	.000	.000
684	683.000	7.091	.000	.000	.000	.000	.000
685	684.000	7.098	.000	.000	.000	.000	.000
686	685.000	7.105	.000	.000	.000	.000	.000
687	686.000	7.113	.000	.000	.000	.000	.000
688	687.000	7.120	.000	.000	.000	.000	.000
689	688.000	7.127	.000	.000	.000	.000	.000
690	689.000	7.135	.000	.000	.000	.000	.000
691	690.000	7.142	.000	.000	.000	.000	.000
692	691.000	7.149	.000	.000	.000	.000	.000
693	692.000	7.157	.000	.000	.000	.000	.000
694	693.000	7.164	.000	.000	.000	.000	.000
695	694.000	7.171	.000	.000	.000	.000	.000
696	695.000	7.179	.000	.000	.000	.000	.000
697	696.000	7.186	.000	.000	.000	.000	.000
698	697.000	7.193	.000	.000	.000	.000	.000
699	698.000	7.200	.000	.000	.000	.000	.000

Figure A3. (Sheet 12 of 15)

700	699.000	7.208	.000	.000	.000	.000	.000
701	700.000	7.215	.000	.000	.000	.000	.000
702	701.000	7.222	.000	.000	.000	.000	.000
703	702.000	7.230	.000	.000	.000	.000	.000
704	703.000	7.237	.000	.000	.000	.000	.000
705	704.000	7.244	.000	.000	.000	.000	.000
706	705.000	7.251	.000	.000	.000	.000	.000
707	706.000	7.259	.000	.000	.000	.000	.000
708	707.000	7.266	.000	.000	.000	.000	.000
709	708.000	7.273	.000	.000	.000	.000	.000
710	709.000	7.280	.000	.000	.000	.000	.000
711	710.000	7.288	.000	.000	.000	.000	.000
712	711.000	7.295	.000	.000	.000	.000	.000
713	712.000	7.302	.000	.000	.000	.000	.000
714	713.000	7.309	.000	.000	.000	.000	.000
715	714.000	7.317	.000	.000	.000	.000	.000
716	715.000	7.324	.000	.000	.000	.000	.000
717	716.000	7.331	.000	.000	.000	.000	.000
718	717.000	7.338	.000	.000	.000	.000	.000
719	718.000	7.345	.000	.000	.000	.000	.000
720	719.000	7.353	.000	.000	.000	.000	.000
721	720.000	7.360	.000	.000	.000	.000	.000
722	721.000	7.367	.000	.000	.000	.000	.000
723	722.000	7.374	.000	.000	.000	.000	.000
724	723.000	7.381	.000	.000	.000	.000	.000
725	724.000	7.389	.000	.000	.000	.000	.000
726	725.000	7.396	.000	.000	.000	.000	.000
727	726.000	7.403	.000	.000	.000	.000	.000
728	727.000	7.410	.000	.000	.000	.000	.000
729	728.000	7.417	.000	.000	.000	.000	.000
730	729.000	7.425	.000	.000	.000	.000	.000
731	730.000	7.432	.000	.000	.000	.000	.000
732	731.000	7.439	.000	.000	.000	.000	.000
733	732.000	7.446	.000	.000	.000	.000	.000
734	733.000	7.453	.000	.000	.000	.000	.000
735	734.000	7.461	.000	.000	.000	.000	.000
736	735.000	7.468	.000	.000	.000	.000	.000
737	736.000	7.475	.000	.000	.000	.000	.000
738	737.000	7.482	.000	.000	.000	.000	.000
739	738.000	7.489	.000	.000	.000	.000	.000
740	739.000	7.496	.000	.000	.000	.000	.000
741	740.000	7.503	.000	.000	.000	.000	.000
742	741.000	7.511	.000	.000	.000	.000	.000
743	742.000	7.518	.000	.000	.000	.000	.000
744	743.000	7.525	.000	.000	.000	.000	.000
745	744.000	7.532	.000	.000	.000	.000	.000
746	745.000	7.539	.000	.000	.000	.000	.000
747	746.000	7.546	.000	.000	.000	.000	.000
748	747.000	7.553	.000	.000	.000	.000	.000
749	748.000	7.561	.000	.000	.000	.000	.000
750	749.000	7.568	.000	.000	.000	.000	.000
751	750.000	7.575	.000	.000	.000	.000	.000
752	751.000	7.582	.000	.000	.000	.000	.000
753	752.000	7.589	.000	.000	.000	.000	.000
754	753.000	7.596	.000	.000	.000	.000	.000
755	754.000	7.603	.000	.000	.000	.000	.000
756	755.000	7.610	.000	.000	.000	.000	.000
757	756.000	7.617	.000	.000	.000	.000	.000
758	757.000	7.624	.000	.000	.000	.000	.000
759	758.000	7.632	.000	.000	.000	.000	.000

Figure A3. (Sheet 13 of 15)

760	759.000	7.639	.000	.000	.000	.000	.000
761	760.000	7.646	.000	.000	.000	.000	.000
762	761.000	7.653	.000	.000	.000	.000	.000
763	762.000	7.660	.000	.000	.000	.000	.000
764	763.000	7.667	.000	.000	.000	.000	.000
765	764.000	7.674	.000	.000	.000	.000	.000
766	765.000	7.681	.000	.000	.000	.000	.000
767	766.000	7.688	.000	.000	.000	.000	.000
768	767.000	7.695	.000	.000	.000	.000	.000
769	768.000	7.702	.000	.000	.000	.000	.000
770	769.000	7.709	.000	.000	.000	.000	.000
771	770.000	7.716	.000	.000	.000	.000	.000
772	771.000	7.723	.000	.000	.000	.000	.000
773	772.000	7.731	.000	.000	.000	.000	.000
774	773.000	7.738	.000	.000	.000	.000	.000
775	774.000	7.745	.000	.000	.000	.000	.000
776	775.000	7.752	.000	.000	.000	.000	.000
777	776.000	7.759	.000	.000	.000	.000	.000
778	777.000	7.766	.000	.000	.000	.000	.000
779	778.000	7.773	.000	.000	.000	.000	.000
780	779.000	7.780	.000	.000	.000	.000	.000

I	DYEQ(I)	VGENV(I)	VUSED(I)	PSV(I)
1	20.000	71.105	55.379	.508
2	21.000	68.445	276.944	.489
3	20.500	69.787	150.520	.498
4	20.000	71.105	55.387	.508
5	20.250	70.445	67.165	.503
6	20.500	69.787	150.504	.498
7	20.375	70.116	106.640	.501
8	20.250	70.445	67.168	.503
9	20.313	70.280	83.041	.502
10	20.281	70.363	72.685	.503
11	20.250	70.445	67.166	.503
12	20.266	70.404	69.207	.503
13	20.281	70.363	72.684	.503
14	20.273	70.383	70.681	.503
15	20.266	70.404	69.208	.503
16	20.270	70.394	69.862	.503
17	20.273	70.383	70.681	.503
18	20.271	70.388	70.251	.503
19	20.272	70.386	70.458	.503
20	20.272	70.387	70.353	.503
21	20.272	70.386	70.408	.503
22	20.272	70.387	70.382	.503
23	20.272	70.387	70.392	.503
24	20.272	70.387	70.387	.503
25	20.272	70.387	70.382	.503
26	20.272	70.387	70.386	.503
27	20.272	70.387	70.387	.503
28	20.272	70.387	70.386	.503
29	20.272	70.387	70.386	.503
30	20.272	70.387	70.387	.503
31	20.272	70.387	70.386	.503
32	20.272	70.387	70.387	.503
33	20.272	70.387	70.387	.503
34	20.272	70.387	70.387	.503
35	20.272	70.387	70.387	.503

Figure A3. (Sheet 14 of 15)

36	20.272	70.387	70.387	.503
37	20.272	70.387	70.387	.503
38	20.272	70.387	70.387	.503
39	20.272	70.387	70.387	.503
40	20.272	70.387	70.387	.503

SOLUTION REACHED, NON-INTERSECTING PROFILES

Figure A3. (Sheet 15 of 15)

```
EXAMPLE 4, CERC REPORT  OCTOBER 23,1991  ] Run Descriptor
 JTYPE IMAX   DY      AO      SO      SP     VADD     SEA    HBERM    HSTAR   HMIX
1   780    1.0    0.10    0.10    0.10   600.0    0.10    -1.5     6.0     0.2

9  •  NUMBER OF INPUT (Y,HO) POINTS ON ORIGINAL PROFILE

  0.0     -1.5    15.0      0.0     90.0      1.0    140.0      2.0 } 9 Pairs of
200.0      3.0   300.0      4.0    415.0      5.0    540.0      6.0 } Distance, Depth
770.0      7.0                                                      } Specifying Initial
                                                                      Profile
 JSED  LMAX   XMU       SIG (Note: XMU, SIG Not Used in This Example)
  1     3    2.64      0.10          JSED, LMAX, & XMU,SIG  in phi units

  0.05   0.035    0.10   0.063     0.15    0.08     0.20    0.100   DI, AI
  0.30   0.120    0.40   0.140     0.50   0.160     0.70    0.170   (19 Pairs)
  0.80   0.200    1.00   0.210      2.0    0.27      5.0     0.36   This Input
  10.0    0.40    20.0    0.49     50.0    0.59    100.0     0.64   Same For
 200.0    0.70   500.0    0.80   1000.0    0.86                     All Runs

  0.6     1.0     0.35     0.5      0.1     0.0   3 Pairs of Diameter, Cumulative
                                                                  Probability
ITSMAX  DYEQ1
 40    100.0
```

Note: Blank Lines Above Have Been Added to Facilitate Annotation.

Figure A4. Listing of input file EQPR.INP for Example 4

EXAMPLE 4, CERC REPORT OCTOBER 23,1991

```
INPUT PAIRS OF DI,AI
   .050     .035     .100      .063     .150      .080     .200      .100
   .300     .120     .400      .140     .500      .160     .700      .170
   .800     .200    1.000      .210    2.000      .270    5.000      .360
 10.000     .400   20.000      .490   50.000      .590  100.000      .640
200.000     .700  500.000      .8001000.000       860
```

```
INPUT PAIRS OF D(mm),P
   .600    1.000     .350      .500     .100      .000
```

I	Y(I)	HO(I)	HP(I)	HEQ(I)	AC(I)	VUSEDC(I)	PCC(I)
1	.000	-1.500	-1.500	.000	.000	.000	.000
2	1.000	-1.400	-1.500	.000	.000	.000	.000
3	2.000	-1.300	-1.500	.000	.000	.000	.000
4	3.000	-1.200	-1.500	.000	.000	.000	.000
5	4.000	-1.100	-1.500	.000	.000	.000	.000
6	5.000	-1.000	-1.500	.000	.000	.000	.000
7	6.000	-.900	-1.500	.000	.000	.000	.000
8	7.000	-.800	-1.500	.000	.000	.000	.000
9	8.000	-.700	-1.500	.000	.000	.000	.000
10	9.000	-.600	-1.500	.000	.000	.000	.000
11	10.000	-.500	-1.500	.000	.000	.000	.000
12	11.000	-.400	-1.500	.000	.000	.000	.000
13	12.000	-.300	-1.500	.000	.000	.000	.000
14	13.000	-.200	-1.500	.000	.000	.000	.000
15	14.000	-.100	-1.500	.000	.000	.000	.000
16	15.000	.000	-1.500	.000	.000	.000	.000
17	16.000	.013	-1.500	.000	.000	.000	.000
18	17.000	.027	-1.500	.000	.000	.000	.000
19	18.000	.040	-1.500	.000	.000	.000	.000
20	19.000	.053	-1.500	.000	.000	.000	.000
21	20.000	.067	-1.500	.000	.000	.000	.000
22	21.000	.080	-1.500	.000	.000	.000	.000
23	22.000	.093	-1.500	.000	.000	.000	.000
24	23.000	.107	-1.500	.000	.000	.000	.000
25	24.000	.120	-1.500	.000	.000	.000	.000
26	25.000	.133	-1.500	.000	.000	.000	.000
27	26.000	.147	-1.500	.000	.000	.000	.000
28	27.000	.160	-1.500	.000	.000	.000	.000
29	28.000	.173	-1.500	.000	.000	.000	.000
30	29.000	.187	-1.500	.000	.000	.000	.000
31	30.000	.200	-1.500	.000	.000	.000	.000
32	31.000	.213	-1.500	.000	.000	.000	.000
33	32.000	.227	-1.500	.000	.000	.000	.000
34	33.000	.240	-1.500	.000	.000	.000	.000
35	34.000	.253	-1.500	.000	.000	.000	.000
36	35.000	.267	-1.500	.000	.000	.000	.000
37	36.000	.280	-1.500	.000	.000	.000	.000
38	37.000	.293	-1.500	.000	.000	.000	.000
39	38.000	.307	-1.500	.000	.000	.000	.000
40	39.000	.320	-1.500	.000	.000	.000	.000
41	40.000	.333	-1.500	.000	.000	.000	.000

Figure A5. Listing of output file EQPR.OUT for Example 4 (Sheet 1 of 15)

42	41.000	.347	-1.500	.000	.000	.000	.000
43	42.000	.360	-1.500	.000	.000	.000	.000
44	43.000	.373	-1.500	.000	.000	.000	.000
45	44.000	.387	-1.500	.000	.000	.000	.000
46	45.000	.400	-1.500	.000	.000	.000	.000
47	46.000	.413	-1.500	.000	.000	.000	.000
48	47.000	.427	-1.500	.000	.000	.000	.000
49	48.000	.440	-1.500	.000	.000	.000	.000
50	49.000	.453	-1.500	.000	.000	.000	.000
51	50.000	.467	-1.500	.000	.000	.000	.000
52	51.000	.480	-1.500	.000	.000	.000	.000
53	52.000	.493	-1.500	.000	.000	.000	.000
54	53.000	.507	-1.500	.000	.000	.000	.000
55	54.000	.520	-1.500	.000	.000	.000	.000
56	55.000	.533	-1.500	.000	.000	.000	.000
57	56.000	.547	-1.500	.000	.000	.000	.000
58	57.000	.560	-1.500	.000	.000	.000	.000
59	58.000	.573	-1.500	.000	.000	.000	.000
60	59.000	.587	-1.500	.000	.000	.000	.000
61	60.000	.600	-1.500	.000	.000	.000	.000
62	61.000	.613	-1.500	.000	.000	.000	.000
63	62.000	.627	-1.500	.000	.000	.000	.000
64	63.000	.640	-1.500	.000	.000	.000	.000
65	64.000	.653	-1.500	.000	.000	.000	.000
66	65.000	.667	-1.500	.000	.000	.000	.000
67	66.000	.680	-1.500	.000	.000	.000	.000
68	67.000	.693	-1.500	.000	.000	.000	.000
69	68.000	.707	-1.500	.000	.000	.000	.000
70	69.000	.720	-1.500	.000	.000	.000	.000
71	70.000	.733	-1.500	.000	.000	.000	.000
72	71.000	.747	-1.500	.000	.000	.000	.000
73	72.000	.760	-1.500	.000	.000	.000	.000
74	73.000	.773	-1.500	.000	.000	.000	.000
75	74.000	.787	-1.500	.000	.000	.000	.000
76	75.000	.800	-1.500	.000	.000	.000	.000
77	76.000	.813	-1.500	.000	.000	.000	.000
78	77.000	.827	-1.500	.000	.000	.000	.000
79	78.000	.840	-1.500	.000	.000	.000	.000
80	79.000	.853	-1.500	.000	.000	.000	.000
81	80.000	.867	-1.500	.000	.000	.000	.000
82	81.000	.880	-1.500	.000	.000	.000	.000
83	82.000	.893	-1.500	.000	.000	.000	.000
84	83.000	.907	-1.500	.000	.000	.000	.000
85	84.000	.920	-1.500	.000	.000	.000	.000
86	85.000	.933	-1.500	.000	.000	.000	.000
87	86.000	.947	-1.500	.000	.000	.000	.000
88	87.000	.960	-1.500	.000	.000	.000	.000
89	88.000	.973	-1.500	.000	.000	.000	.000
90	89.000	.987	-1.500	.000	.000	.000	.000
91	90.000	1.000	-1.500	.000	.000	.000	.000
92	91.000	1.020	-1.500	.000	.000	.000	.000
93	92.000	1.040	-1.500	.000	.000	.000	.000
94	93.000	1.060	-1.500	.000	.000	.000	.000
95	94.000	1.080	-1.500	.000	.000	.000	.000
96	95.000	1.100	-1.500	.000	.000	.000	.000
97	96.000	1.120	-1.500	.000	.000	.000	.000
98	97.000	1.140	-1.500	.000	.000	.000	.000
99	98.000	1.160	-1.500	.000	.000	.000	.000
100	99.000	1.180	-1.500	.000	.000	.000	.000
101	100.000	1.200	-1.500	.000	.000	.000	.000

Figure A5. (Sheet 2 of 15)

102	101.000	1.220	-1.500	.000	.000	.000	.000
103	102.000	1.240	-1.500	.000	.000	.000	.000
104	103.000	1.260	-1.500	.000	.000	.000	.000
105	104.000	1.280	-1.500	.000	.000	.000	.000
106	105.000	1.300	-1.500	.000	.000	.000	.000
107	106.000	1.320	-1.500	.000	.000	.000	.000
108	107.000	1.340	-1.500	.000	.000	.000	.000
109	108.000	1.360	-1.500	.000	.000	.000	.000
110	109.000	1.380	-1.500	.000	.000	.000	.000
111	110.000	1.400	-1.500	.000	.000	.000	.000
112	111.000	1.420	-1.500	.000	.000	.000	.000
113	112.000	1.440	-1.500	.000	.000	.000	.000
114	113.000	1.460	-1.500	.000	.000	.000	.000
115	114.000	1.480	-1.500	.000	.000	.000	.000
116	115.000	1.500	-1.500	.000	.000	.000	.000
117	116.000	1.520	-1.500	.000	.000	.000	.000
118	117.000	1.540	-1.500	.000	.000	.000	.000
119	118.000	1.560	-1.500	.000	.000	.000	.000
120	119.000	1.580	-1.500	.000	.000	.000	.000
121	120.000	1.600	-1.500	.000	.000	.000	.000
122	121.000	1.620	-1.500	.000	.000	.000	.000
123	122.000	1.640	-1.500	.000	.000	.000	.000
124	123.000	1.660	-1.500	.000	.000	.000	.000
125	124.000	1.680	-1.500	.000	.000	.000	.000
126	125.000	1.700	-1.500	.000	.000	.000	.000
127	126.000	1.720	-1.500	.000	.000	.000	.000
128	127.000	1.740	-1.500	.000	.000	.000	.000
129	128.000	1.760	-1.500	.000	.000	.000	.000
130	129.000	1.780	-1.500	.000	.000	.000	.000
131	130.000	1.800	-1.500	.000	.000	.000	.000
132	131.000	1.820	-1.500	.000	.000	.000	.000
133	132.000	1.840	-1.500	.000	.000	.000	.000
134	133.000	1.860	-1.500	.000	.000	.000	.000
135	134.000	1.880	-1.500	.000	.000	.000	.000
136	135.000	1.900	-1.500	.000	.000	.000	.000
137	136.000	1.920	-1.500	.000	.000	.000	.000
138	137.000	1.940	-1.500	.000	.000	.000	.000
139	138.000	1.960	-1.500	.000	.000	.000	.000
140	139.000	1.980	-1.500	.000	.000	.000	.000
141	140.000	2.000	-1.500	-1.500	.000	.000	.000
142	141.000	2.017	-1.500	-1.370	.000	.000	.000
143	142.000	2.033	-1.500	-1.270	.000	.000	.000
144	143.000	2.050	-1.500	-1.170	.000	.000	.000
145	144.000	2.067	-1.500	-1.070	.000	.000	.000
146	145.000	2.083	-1.500	-.970	.000	.000	.000
147	146.000	2.100	-1.500	-.870	.000	.000	.000
148	147.000	2.117	-1.500	-.770	.000	.000	.000
149	148.000	2.133	-1.500	-.670	.000	.000	.000
150	149.000	2.150	-1.500	-.570	.000	.000	.000
151	150.000	2.167	-1.500	-.470	.000	.000	.000
152	151.000	2.183	-1.500	-.370	.000	.000	.000
153	152.000	2.200	-1.500	-.270	.000	.000	.000
154	153.000	2.217	-1.500	-.170	.000	.000	.000
155	154.000	2.233	-1.500	-.070	.164	3.060	.021
156	155.000	2.250	-1.500	.024	.163	3.260	.022
157	156.000	2.267	-1.500	.089	.163	3.460	.023
158	157.000	2.283	-1.500	.145	.163	3.660	.025
159	158.000	2.300	-1.500	.197	.163	3.860	.026
160	159.000	2.317	-1.500	.245	.163	4.060	.027
161	160.000	2.333	-1.500	.291	.163	4.260	.029

Figure A5. (Sheet 3 of 15)

162	161.000	2.350	-1.500	.335	.163	4.460	.030
163	162.000	2.367	-1.500	.377	.163	4.660	.031
164	163.000	2.383	-1.500	.418	.163	4.860	.033
165	164.000	2.400	-1.500	.458	.163	5.060	.034
166	165.000	2.417	-1.500	.497	.163	5.260	.036
167	166.000	2.433	-1.500	.535	.162	5.460	.037
168	167.000	2.450	-1.500	.572	.162	5.660	.038
169	168.000	2.467	-1.500	.608	.162	5.860	.040
170	169.000	2.483	-1.500	.643	.162	6.060	.041
171	170.000	2.500	-1.500	.678	.162	6.260	.042
172	171.000	2.517	-1.500	.712	.162	6.460	.044
173	172.000	2.533	-1.500	.746	.162	6.660	.045
174	173.000	2.550	-1.500	.779	.162	6.860	.046
175	174.000	2.567	-1.500	.812	.162	7.060	.048
176	175.000	2.583	-1.500	.844	.162	7.260	.049
177	176.000	2.600	-1.500	.876	.161	7.460	.050
178	177.000	2.617	-1.500	.908	.161	7.660	.052
179	178.000	2.633	-1.500	.939	.161	7.860	.053
180	179.000	2.650	-1.500	.969	.161	8.060	.054
181	180.000	2.667	-1.500	1.000	.161	8.260	.056
182	181.000	2.683	-1.500	1.030	.161	8.460	.057
183	182.000	2.700	-1.500	1.059	.161	8.660	.058
184	183.000	2.717	-1.500	1.089	.161	8.860	.060
185	184.000	2.733	-1.500	1.118	.161	9.060	.061
186	185.000	2.750	-1.500	1.146	.161	9.260	.063
187	186.000	2.767	-1.500	1.175	.161	9.460	.064
188	187.000	2.783	-1.500	1.203	.160	9.660	.065
189	188.000	2.800	-1.406	1.231	.160	9.860	.067
190	189.000	2.817	-1.306	1.259	.160	10.060	.068
191	190.000	2.833	-1.206	1.286	.160	10.260	.069
192	191.000	2.850	-1.106	1.313	.160	10.460	.071
193	192.000	2.867	-1.006	1.341	.160	10.660	.072
194	193.000	2.883	-.906	1.367	.160	10.860	.073
195	194.000	2.900	-.806	1.394	.160	11.060	.075
196	195.000	2.917	-.706	1.420	.160	11.260	.076
197	196.000	2.933	-.606	1.447	.160	11.460	.077
198	197.000	2.950	-.506	1.473	.159	11.660	.079
199	198.000	2.967	-.406	1.498	.159	11.860	.080
200	199.000	2.983	-.306	1.524	.159	12.060	.081
201	200.000	3.000	-.206	1.550	.159	12.260	.083
202	201.000	3.010	-.106	1.575	.159	12.460	.084
203	202.000	3.020	-.006	1.600	.159	12.660	.086
204	203.000	3.030	.094	1.625	.159	12.860	.087
205	204.000	3.040	.194	1.650	.159	13.060	.088
206	205.000	3.050	.294	1.674	.159	13.260	.090
207	206.000	3.060	.394	1.699	.159	13.460	.091
208	207.000	3.070	.494	1.723	.159	13.660	.092
209	208.000	3.080	.594	1.748	.158	13.860	.094
210	209.000	3.090	.694	1.772	.158	14.060	.095
211	210.000	3.100	.794	1.796	.158	14.260	.096
212	211.000	3.110	.894	1.819	.158	14.460	.098
213	212.000	3.120	.994	1.843	.158	14.660	.099
214	213.000	3.130	1.094	1.867	.158	14.860	.100
215	214.000	3.140	1.194	1.890	.158	15.060	.102
216	215.000	3.150	1.294	1.913	.158	15.260	.103
217	216.000	3.160	1.394	1.936	.158	15.460	.104
218	217.000	3.170	1.494	1.959	.158	15.660	.106
219	218.000	3.180	1.594	1.982	.158	15.860	.107
220	219.000	3.190	1.694	2.005	.157	16.060	.108
221	220.000	3.200	1.794	2.028	.157	16.260	.110

Figure A5. (Sheet 4 of 15)

222	221.000	3.210	1.894	2.050	.157	16.460	.111
223	222.000	3.220	1.994	2.073	.157	16.660	.113
224	223.000	3.230	2.094	2.095	.157	16.860	.114
225	224.000	3.240	2.194	2.117	.157	17.101	.116
226	225.000	3.250	2.294	2.139	.157	17.216	.116
227	226.000	3.260	2.394	2.161	.157	17.409	.118
228	227.000	3.270	2.494	2.183	.157	17.680	.119
229	228.000	3.280	2.594	2.205	.156	18.030	.122
230	229.000	3.290	2.694	2.227	.156	18.457	.125
231	230.000	3.300	2.794	2.248	.156	18.963	.128
232	231.000	3.310	2.894	2.270	.156	19.547	.132
233	232.000	3.320	2.994	2.291	.155	20.210	.137
234	233.000	3.330	3.094	2.313	.155	20.952	.142
235	234.000	3.340	3.194	2.334	.155	21.772	.147
236	235.000	3.350	3.294	2.355	.154	22.672	.153
237	236.000	3.360	3.394	2.375	.154	23.435	.158
238	237.000	3.370	.000	2.396	.153	24.414	.165
239	238.000	3.380	.000	2.417	.153	25.383	.171
240	239.000	3.390	.000	2.437	.153	26.341	.178
241	240.000	3.400	.000	2.457	.152	27.289	.184
242	241.000	3.410	.000	2.477	.152	28.227	.191
243	242.000	3.420	.000	2.497	.151	29.155	.197
244	243.000	3.430	.000	2.517	.151	30.072	.203
245	244.000	3.440	.000	2.537	.150	30.980	.209
246	245.000	3.450	.000	2.556	.150	31.879	.215
247	246.000	3.460	.000	2.576	.150	32.768	.221
248	247.000	3.470	.000	2.595	.149	33.647	.227
249	248.000	3.480	.000	2.614	.149	34.517	.233
250	249.000	3.490	.000	2.634	.148	35.378	.239
251	250.000	3.500	.000	2.652	.148	36.230	.245
252	251.000	3.510	.000	2.671	.147	37.074	.250
253	252.000	3.520	.000	2.690	.147	37.908	.256
254	253.000	3.530	.000	2.709	.147	38.733	.262
255	254.000	3.540	.000	2.727	.146	39.550	.267
256	255.000	3.550	.000	2.746	.146	40.359	.273
257	256.000	3.560	.000	2.764	.146	41.159	.278
258	257.000	3.570	.000	2.782	.145	41.951	.283
259	258.000	3.580	.000	2.800	.145	42.735	.289
260	259.000	3.590	.000	2.818	.144	43.511	.294
261	260.000	3.600	.000	2.836	.144	44.279	.299
262	261.000	3.610	.000	2.854	.144	45.039	.304
263	262.000	3.620	.000	2.871	.143	45.791	.309
264	263.000	3.630	.000	2.889	.143	46.536	.314
265	264.000	3.640	.000	2.907	.143	47.273	.319
266	265.000	3.650	.000	2.924	.142	48.003	.324
267	266.000	3.660	.000	2.941	.142	48.726	.329
268	267.000	3.670	.000	2.958	.142	49.441	.334
269	268.000	3.680	.000	2.975	.141	50.149	.339
270	269.000	3.690	.000	2.992	.141	50.850	.343
271	270.000	3.700	.000	3.009	.141	51.544	.348
272	271.000	3.710	.000	3.026	.140	52.231	.353
273	272.000	3.720	.000	3.043	.140	52.912	.357
274	273.000	3.730	.000	3.060	.140	53.585	.362
275	274.000	3.740	.000	3.076	.139	54.252	.366
276	275.000	3.750	.000	3.093	.139	54.913	.371
277	276.000	3.760	.000	3.109	.139	55.567	.375
278	277.000	3.770	.000	3.125	.138	56.215	.380
279	278.000	3.780	.000	3.142	.138	56.856	.384
280	279.000	3.790	.000	3.158	.138	57.491	.388
281	280.000	3.800	.000	3.174	.138	58.120	.393

Figure A5. (Sheet 5 of 15)

282	281.000	3.810	.000	3.190	.137	58.743	.397
283	282.000	3.820	.000	3.206	.137	59.361	.401
284	283.000	3.830	.000	3.222	.137	59.972	.405
285	284.000	3.840	.000	3.238	.136	60.577	.409
286	285.000	3.850	.000	3.253	.136	61.177	.413
287	286.000	3.860	.000	3.269	.136	61.771	.417
288	287.000	3.870	.000	3.284	.136	62.359	.421
289	288.000	3.880	.000	3.300	.135	62.942	.425
290	289.000	3.890	.000	3.315	.135	63.519	.429
291	290.000	3.900	.000	3.331	.135	64.091	.433
292	291.000	3.910	.000	3.346	.134	64.658	.437
293	292.000	3.920	.000	3.361	.134	65.219	.440
294	293.000	3.930	.000	3.376	.134	65.776	.444
295	294.000	3.940	.000	3.391	.134	66.327	.448
296	295.000	3.950	.000	3.406	.133	66.873	.452
297	296.000	3.960	.000	3.421	.133	67.414	.455
298	297.000	3.970	.000	3.436	.133	67.950	.459
299	298.000	3.980	.000	3.451	.133	68.482	.463
300	299.000	3.990	.000	3.466	.132	69.008	.466
301	300.000	4.000	.000	3.481	.132	69.530	.470
302	301.000	4.009	.000	3.495	.132	70.047	.473
303	302.000	4.017	.000	3.510	.132	70.557	.477
304	303.000	4.026	.000	3.524	.131	71.062	.480
305	304.000	4.035	.000	3.539	.131	71.561	.483
306	305.000	4.043	.000	3.553	.131	72.054	.487
307	306.000	4.052	.000	3.567	.131	72.542	.490
308	307.000	4.061	.000	3.582	.130	73.024	.493
309	308.000	4.070	.000	3.596	.130	73.500	.496
310	309.000	4.078	.000	3.610	.130	73.971	.500
311	310.000	4.087	.000	3.624	.130	74.436	.503
312	311.000	4.096	.000	3.638	.129	74.896	.506
313	312.000	4.104	.000	3.652	.129	75.351	.509
314	313.000	4.113	.000	3.666	.128	75.801	.512
315	314.000	4.122	.000	3.680	.128	76.245	.515
316	315.000	4.130	.000	3.694	.128	76.684	.518
317	316.000	4.139	.000	3.707	.127	77.119	.521
318	317.000	4.148	.000	3.721	.127	77.548	.524
319	318.000	4.157	.000	3.734	.126	77.973	.527
320	319.000	4.165	.000	3.748	.126	78.393	.529
321	320.000	4.174	.000	3.761	.126	78.808	.532
322	321.000	4.183	.000	3.774	.125	79.218	.535
323	322.000	4.191	.000	3.788	.125	79.624	.538
324	323.000	4.200	.000	3.801	.125	80.026	.541
325	324.000	4.209	.000	3.814	.124	80.423	.543
326	325.000	4.217	.000	3.827	.124	80.816	.546
327	326.000	4.226	.000	3.840	.124	81.204	.548
328	327.000	4.235	.000	3.853	.123	81.589	.551
329	328.000	4.243	.000	3.865	.123	81.969	.554
330	329.000	4.252	.000	3.878	.122	82.345	.556
331	330.000	4.261	.000	3.891	.122	82.717	.559
332	331.000	4.270	.000	3.903	.122	83.086	.561
333	332.000	4.278	.000	3.916	.121	83.450	.564
334	333.000	4.287	.000	3.928	.121	83.810	.566
335	334.000	4.296	.000	3.941	.121	84.167	.568
336	335.000	4.304	.000	3.953	.121	84.520	.571
337	336.000	4.313	.000	3.965	.120	84.870	.573
338	337.000	4.322	.000	3.978	.120	85.216	.576
339	338.000	4.330	.000	3.990	.120	85.558	.578
340	339.000	4.339	.000	4.002	.119	85.897	.580
341	340.000	4.348	.000	4.014	.119	86.233	.582

Figure A5. (Sheet 6 of 15)

342	341.000	4.357	.000	4.026	.119	86.565	.585
343	342.000	4.365	.000	4.038	.118	86.894	.587
344	343.000	4.374	.000	4.050	.118	87.220	.589
345	344.000	4.383	.000	4.062	.118	87.542	.591
346	345.000	4.391	.000	4.073	.117	87.862	.593
347	346.000	4.400	.000	4.085	.117	88.178	.596
348	347.000	4.409	.000	4.097	.117	88.491	.598
349	348.000	4.417	.000	4.108	.117	88.802	.600
350	349.000	4.426	.000	4.120	.116	89.109	.602
351	350.000	4.435	.000	4.132	.116	89.414	.604
352	351.000	4.443	.000	4.143	.116	89.716	.606
353	352.000	4.452	.000	4.154	.116	90.015	.608
354	353.000	4.461	.000	4.166	.115	90.311	.610
355	354.000	4.470	.000	4.177	.115	90.605	.612
356	355.000	4.478	.000	4.188	.115	90.896	.614
357	356.000	4.487	.000	4.200	.114	91.185	.616
358	357.000	4.496	.000	4.211	.114	91.471	.618
359	358.000	4.504	.000	4.222	.114	91.754	.620
360	359.000	4.513	.000	4.233	.114	92.035	.622
361	360.000	4.522	.000	4.244	.113	92.314	.623
362	361.000	4.530	.000	4.255	.113	92.591	.625
363	362.000	4.539	.000	4.266	.113	92.865	.627
364	363.000	4.548	.000	4.277	.113	93.137	.629
365	364.000	4.557	.000	4.288	.112	93.407	.631
366	365.000	4.565	.000	4.299	.112	93.674	.633
367	366.000	4.574	.000	4.309	.112	93.940	.634
368	367.000	4.583	.000	4.320	.112	94.203	.636
369	368.000	4.591	.000	4.331	.112	94.465	.638
370	369.000	4.600	.000	4.342	.111	94.724	.640
371	370.000	4.609	.000	4.352	.111	94.982	.642
372	371.000	4.617	.000	4.363	.111	95.237	.643
373	372.000	4.626	.000	4.373	.111	95.491	.645
374	373.000	4.635	.000	4.384	.110	95.743	.647
375	374.000	4.643	.000	4.394	.110	95.993	.648
376	375.000	4.652	.000	4.405	.110	96.242	.650
377	376.000	4.661	.000	4.415	.110	96.488	.652
378	377.000	4.670	.000	4.425	.109	96.734	.653
379	378.000	4.678	.000	4.436	.109	96.977	.655
380	379.000	4.687	.000	4.446	.109	97.219	.657
381	380.000	4.696	.000	4.456	.109	97.459	.658
382	381.000	4.704	.000	4.466	.109	97.698	.660
383	382.000	4.713	.000	4.476	.108	97.936	.661
384	383.000	4.722	.000	4.486	.108	98.172	.663
385	384.000	4.730	.000	4.496	.108	98.406	.665
386	385.000	4.739	.000	4.507	.108	98.640	.666
387	386.000	4.748	.000	4.516	.108	98.872	.668
388	387.000	4.757	.000	4.526	.107	99.102	.669
389	388.000	4.765	.000	4.536	.107	99.332	.671
390	389.000	4.774	.000	4.546	.107	99.560	.672
391	390.000	4.783	.000	4.556	.107	99.787	.674
392	391.000	4.791	.000	4.566	.106	100.013	.675
393	392.000	4.800	.000	4.576	.106	100.238	.677
394	393.000	4.809	.000	4.585	.106	100.462	.679
395	394.000	4.817	.000	4.595	.106	100.685	.680
396	395.000	4.826	.000	4.605	.106	100.906	.682
397	396.000	4.835	.000	4.614	.105	101.127	.683
398	397.000	4.843	.000	4.624	.105	101.347	.685
399	398.000	4.852	.000	4.634	.105	101.566	.686
400	399.000	4.861	.000	4.643	.105	101.784	.687
401	400.000	4.870	.000	4.653	.105	102.001	.689

Figure A5. (Sheet 7 of 15)

402	401.000	4.878	.000	4.662	.104	102.218	.690
403	402.000	4.887	.000	4.672	.104	102.434	.692
404	403.000	4.896	.000	4.681	.104	102.649	.693
405	404.000	4.904	.000	4.690	.104	102.863	.695
406	405.000	4.913	.000	4.700	.104	103.077	.696
407	406.000	4.922	.000	4.709	.104	103.290	.698
408	407.000	4.930	.000	4.718	.103	103.502	.699
409	408.000	4.939	.000	4.728	.103	103.714	.700
410	409.000	4.948	.000	4.737	.103	103.925	.702
411	410.000	4.957	.000	4.746	.103	104.136	.703
412	411.000	4.965	.000	4.755	.103	104.347	.705
413	412.000	4.974	.000	4.764	.102	104.557	.706
414	413.000	4.983	.000	4.773	.102	104.766	.708
415	414.000	4.991	.000	4.782	.102	104.975	.709
416	415.000	5.000	.000	4.791	.102	105.184	.710
417	416.000	5.008	.000	4.800	.102	105.392	.712
418	417.000	5.016	.000	4.809	.101	105.599	.713
419	418.000	5.024	.000	4.818	.101	105.805	.715
420	419.000	5.032	.000	4.827	.101	106.011	.716
421	420.000	5.040	.000	4.836	.101	106.215	.717
422	421.000	5.048	.000	4.845	.101	106.419	.719
423	422.000	5.056	.000	4.854	.101	106.621	.720
424	423.000	5.064	.000	4.863	.100	106.823	.721
425	424.000	5.072	.000	4.871	.100	107.024	.723
426	425.000	5.080	.000	4.880	.100	107.225	.724
427	426.000	5.088	.000	4.889	.100	107.424	.726
428	427.000	5.096	.000	4.897	.100	107.623	.727
429	428.000	5.104	.000	4.906	.099	107.821	.728
430	429.000	5.112	.000	4.915	.099	108.019	.730
431	430.000	5.120	.000	4.923	.099	108.216	.731
432	431.000	5.128	.000	4.932	.099	108.413	.732
433	432.000	5.136	.000	4.940	.099	108.608	.734
434	433.000	5.144	.000	4.949	.099	108.804	.735
435	434.000	5.152	.000	4.957	.098	108.999	.736
436	435.000	5.160	.000	4.966	.098	109.193	.737
437	436.000	5.168	.000	4.974	.098	109.387	.739
438	437.000	5.176	.000	4.983	.098	109.581	.740
439	438.000	5.184	.000	4.991	.098	109.774	.741
440	439.000	5.192	.000	4.999	.097	109.967	.743
441	440.000	5.200	.000	5.008	.097	110.159	.744
442	441.000	5.208	.000	5.016	.097	110.352	.745
443	442.000	5.216	.000	5.024	.097	110.544	.747
444	443.000	5.224	.000	5.032	.097	110.735	.748
445	444.000	5.232	.000	5.041	.097	110.927	.749
446	445.000	5.240	.000	5.049	.096	111.118	.750
447	446.000	5.248	.000	5.057	.096	111.309	.752
448	447.000	5.256	.000	5.065	.096	111.500	.753
449	448.000	5.264	.000	5.073	.096	111.691	.754
450	449.000	5.272	.000	5.081	.096	111.882	.756
451	450.000	5.280	.000	5.089	.096	112.072	.757
452	451.000	5.288	.000	5.097	.095	112.263	.758
453	452.000	5.296	.000	5.105	.095	112.454	.760
454	453.000	5.304	.000	5.113	.095	112.644	.761
455	454.000	5.312	.000	5.121	.095	112.835	.762
456	455.000	5.320	.000	5.129	.095	113.026	.763
457	456.000	5.328	.000	5.137	.095	113.217	.765
458	457.000	5.336	.000	5.145	.094	113.408	.766
459	458.000	5.344	.000	5.153	.094	113.599	.767
460	459.000	5.352	.000	5.161	.094	113.790	.769
461	460.000	5.360	.000	5.168	.094	113.981	.770

Figure A5. (Sheet 8 of 15)

462	461.000	5.368	.000	5.176	.094	114.173	.771
463	462.000	5.376	.000	5.184	.093	114.365	.772
464	463.000	5.384	.000	5.192	.093	114.557	.774
465	464.000	5.392	.000	5.199	.093	114.750	.775
466	465.000	5.400	.000	5.207	.093	114.942	.776
467	466.000	5.408	.000	5.215	.093	115.135	.778
468	467.000	5.416	.000	5.222	.093	115.329	.779
469	468.000	5.424	.000	5.230	.092	115.523	.780
470	469.000	5.432	.000	5.237	.092	115.717	.782
471	470.000	5.440	.000	5.245	.092	115.912	.783
472	471.000	5.448	.000	5.253	.092	116.107	.784
473	472.000	5.456	.000	5.260	.092	116.303	.786
474	473.000	5.464	.000	5.268	.092	116.499	.787
475	474.000	5.472	.000	5.275	.091	116.696	.788
476	475.000	5.480	.000	5.282	.091	116.893	.790
477	476.000	5.488	.000	5.290	.091	117.091	.791
478	477.000	5.496	.000	5.297	.091	117.289	.792
479	478.000	5.504	.000	5.305	.091	117.489	.794
480	479.000	5.512	.000	5.312	.090	117.688	.795
481	480.000	5.520	.000	5.319	.090	117.889	.796
482	481.000	5.528	.000	5.326	.090	118.090	.798
483	482.000	5.536	.000	5.334	.090	118.292	.799
484	483.000	5.544	.000	5.341	.090	118.495	.800
485	484.000	5.552	.000	5.348	.090	118.698	.802
486	485.000	5.560	.000	5.355	.089	118.903	.803
487	486.000	5.568	.000	5.362	.089	119.108	.804
488	487.000	5.576	.000	5.370	.089	119.314	.806
489	488.000	5.584	.000	5.377	.089	119.521	.807
490	489.000	5.592	.000	5.384	.089	119.729	.809
491	490.000	5.600	.000	5.391	.088	119.938	.810
492	491.000	5.608	.000	5.398	.088	120.147	.811
493	492.000	5.616	.000	5.405	.088	120.358	.813
494	493.000	5.624	.000	5.412	.088	120.570	.814
495	494.000	5.632	.000	5.419	.088	120.783	.816
496	495.000	5.640	.000	5.426	.087	120.997	.817
497	496.000	5.648	.000	5.432	.087	121.212	.819
498	497.000	5.656	.000	5.439	.087	121.428	.820
499	498.000	5.664	.000	5.446	.087	121.645	.822
500	499.000	5.672	.000	5.453	.087	121.863	.823
501	500.000	5.680	.000	5.460	.087	122.083	.825
502	501.000	5.688	.000	5.467	.086	122.304	.826
503	502.000	5.696	.000	5.473	.086	122.526	.828
504	503.000	5.704	.000	5.480	.086	122.749	.829
505	504.000	5.712	.000	5.487	.086	122.974	.831
506	505.000	5.720	.000	5.493	.085	123.200	.832
507	506.000	5.728	.000	5.500	.085	123.427	.834
508	507.000	5.736	.000	5.507	.085	123.656	.835
509	508.000	5.744	.000	5.513	.085	123.886	.837
510	509.000	5.752	.000	5.520	.085	124.117	.838
511	510.000	5.760	.000	5.526	.084	124.350	.840
512	511.000	5.768	.000	5.533	.084	124.585	.841
513	512.000	5.776	.000	5.539	.084	124.821	.843
514	513.000	5.784	.000	5.546	.084	125.058	.845
515	514.000	5.792	.000	5.552	.084	125.297	.846
516	515.000	5.800	.000	5.559	.083	125.538	.848
517	516.000	5.808	.000	5.565	.083	125.780	.850
518	517.000	5.816	.000	5.571	.083	126.024	.851
519	518.000	5.824	.000	5.578	.083	126.270	.853
520	519.000	5.832	.000	5.584	.082	126.517	.855
521	520.000	5.840	.000	5.590	.082	126.766	.856

Figure A5. (Sheet 9 of 15)

522	521.000	5.848	.000	5.596	.082	127.017	.858
523	522.000	5.856	.000	5.603	.082	127.269	.860
524	523.000	5.864	.000	5.609	.082	127.524	.861
525	524.000	5.872	.000	5.615	.081	127.780	.863
526	525.000	5.880	.000	5.621	.081	128.038	.865
527	526.000	5.888	.000	5.627	.081	128.298	.867
528	527.000	5.896	.000	5.633	.081	128.560	.868
529	528.000	5.904	.000	5.639	.080	128.823	.870
530	529.000	5.912	.000	5.645	.080	129.089	.872
531	530.000	5.920	.000	5.651	.080	129.357	.874
532	531.000	5.928	.000	5.657	.080	129.627	.876
533	532.000	5.936	.000	5.663	.079	129.898	.877
534	533.000	5.944	.000	5.669	.079	130.172	.879
535	534.000	5.952	.000	5.675	.079	130.448	.881
536	535.000	5.960	.000	5.681	.079	130.726	.883
537	536.000	5.968	.000	5.687	.078	131.007	.885
538	537.000	5.976	.000	5.692	.078	131.289	.887
539	538.000	5.984	.000	5.698	.078	131.574	.889
540	539.000	5.992	.000	5.704	.078	131.861	.891
541	540.000	6.000	.000	5.710	.077	132.150	.893
542	541.000	6.004	.000	5.715	.077	132.440	.895
543	542.000	6.009	.000	5.721	.077	132.729	.896
544	543.000	6.013	.000	5.726	.077	133.016	.898
545	544.000	6.017	.000	5.732	.076	133.302	.900
546	545.000	6.022	.000	5.738	.076	133.587	.902
547	546.000	6.026	.000	5.743	.076	133.870	.904
548	547.000	6.030	.000	5.749	.076	134.153	.906
549	548.000	6.035	.000	5.754	.075	134.434	.908
550	549.000	6.039	.000	5.759	.075	134.714	.910
551	550.000	6.043	.000	5.765	.075	134.993	.912
552	551.000	6.048	.000	5.770	.075	135.271	.914
553	552.000	6.052	.000	5.776	.074	135.549	.916
554	553.000	6.057	.000	5.781	.074	135.825	.917
555	554.000	6.061	.000	5.786	.074	136.100	.919
556	555.000	6.065	.000	5.791	.074	136.374	.921
557	556.000	6.070	.000	5.797	.073	136.647	.923
558	557.000	6.074	.000	5.802	.073	136.920	.925
559	558.000	6.078	.000	5.807	.073	137.191	.927
560	559.000	6.083	.000	5.812	.073	137.462	.928
561	560.000	6.087	.000	5.817	.072	137.732	.930
562	561.000	6.091	.000	5.823	.072	138.001	.932
563	562.000	6.096	.000	5.828	.072	138.270	.934
564	563.000	6.100	.000	5.833	.072	138.537	.936
565	564.000	6.104	.000	5.838	.071	138.804	.937
566	565.000	6.109	.000	5.843	.071	139.071	.939
567	566.000	6.113	.000	5.848	.071	139.336	.941
568	567.000	6.117	.000	5.853	.071	139.602	.943
569	568.000	6.122	.000	5.857	.070	139.866	.945
570	569.000	6.126	.000	5.862	.070	140.130	.946
571	570.000	6.130	.000	5.867	.070	140.393	.948
572	571.000	6.135	.000	5.872	.070	140.656	.950
573	572.000	6.139	.000	5.877	.069	140.919	.952
574	573.000	6.143	.000	5.882	.069	141.181	.954
575	574.000	6.148	.000	5.886	.069	141.442	.955
576	575.000	6.152	.000	5.891	.069	141.704	.957
577	576.000	6.157	.000	5.896	.069	141.964	.959
578	577.000	6.161	.000	5.901	.068	142.225	.961
579	578.000	6.165	.000	5.905	.068	142.485	.962
580	579.000	6.170	.000	5.910	.068	142.745	.964
581	580.000	6.174	.000	5.915	.068	143.004	.966

Figure A5. (Sheet 10 of 15)

582	581.000	6.178	.000	5.919	.067	143.263	.968
583	582.000	6.183	.000	5.924	.067	143.522	.969
584	583.000	6.187	.000	5.928	.067	143.781	.971
585	584.000	6.191	.000	5.933	.067	144.040	.973
586	585.000	6.196	.000	5.937	.066	144.298	.975
587	586.000	6.200	.000	5.942	.066	144.557	.976
588	587.000	6.204	.000	5.946	.066	144.815	.978
589	588.000	6.209	.000	5.951	.066	145.073	.980
590	589.000	6.213	.000	5.955	.065	145.331	.982
591	590.000	6.217	.000	5.959	.065	145.589	.983
592	591.000	6.222	.000	5.964	.065	145.847	.985
593	592.000	6.226	.000	5.968	.065	146.105	.987
594	593.000	6.230	.000	5.972	.065	146.363	.989
595	594.000	6.235	.000	5.977	.064	146.621	.990
596	595.000	6.239	.000	5.981	.064	146.879	.992
597	596.000	6.243	.000	5.985	.064	147.138	.994
598	597.000	6.248	.000	5.989	.064	147.396	.996
599	598.000	6.252	.000	5.994	.063	147.655	.997
600	599.000	6.257	.000	5.998	.063	147.913	.999
601	600.000	6.261	.000	6.002	.063	148.058	1.000
602	601.000	6.265	.000	6.006	.000	.000	.000
603	602.000	6.270	.000	.000	.000	.000	.000
604	603.000	6.274	.000	.000	.000	.000	.000
605	604.000	6.278	.000	.000	.000	.000	.000
606	605.000	6.283	.000	.000	.000	.000	.000
607	606.000	6.287	.000	.000	.000	.000	.000
608	607.000	6.291	.000	.000	.000	.000	.000
609	608.000	6.296	.000	.000	.000	.000	.000
610	609.000	6.300	.000	.000	.000	.000	.000
611	610.000	6.304	.000	.000	.000	.000	.000
612	611.000	6.309	.000	.000	.000	.000	.000
613	612.000	6.313	.000	.000	.000	.000	.000
614	613.000	6.317	.000	.000	.000	.000	.000
615	614.000	6.322	.000	.000	.000	.000	.000
616	615.000	6.326	.000	.000	.000	.000	.000
617	616.000	6.330	.000	.000	.000	.000	.000
618	617.000	6.335	.000	.000	.000	.000	.000
619	618.000	6.339	.000	.000	.000	.000	.000
620	619.000	6.343	.000	.000	.000	.000	.000
621	620.000	6.348	.000	.000	.000	.000	.000
622	621.000	6.352	.000	.000	.000	.000	.000
623	622.000	6.357	.000	.000	.000	.000	.000
624	623.000	6.361	.000	.000	.000	.000	.000
625	624.000	6.365	.000	.000	.000	.000	.000
626	625.000	6.370	.000	.000	.000	.000	.000
627	626.000	6.374	.000	.000	.000	.000	.000
628	627.000	6.378	.000	.000	.000	.000	.000
629	628.000	6.383	.000	.000	.000	.000	.000
630	629.000	6.387	.000	.000	.000	.000	.000
631	630.000	6.391	.000	.000	.000	.000	.000
632	631.000	6.396	.000	.000	.000	.000	.000
633	632.000	6.400	.000	.000	.000	.000	.000
634	633.000	6.404	.000	.000	.000	.000	.000
635	634.000	6.409	.000	.000	.000	.000	.000
636	635.000	6.413	.000	.000	.000	.000	.000
637	636.000	6.417	.000	.000	.000	.000	.000
638	637.000	6.422	.000	.000	.000	.000	.000
639	638.000	6.426	.000	.000	.000	.000	.000
640	639.000	6.430	.000	.000	.000	.000	.000
641	640.000	6.435	.000	.000	.000	.000	.000

Figure A5. (Sheet 11 of 15)

642	641.000	6.439	.000	.000	.000	.000	.000
643	642.000	6.443	.000	.000	.000	.000	.000
644	643.000	6.448	.000	.000	.000	.000	.000
645	644.000	6.452	.000	.000	.000	.000	.000
646	645.000	6.457	.000	.000	.000	.000	.000
647	646.000	6.461	.000	.000	.000	.000	.000
648	647.000	6.465	.000	.000	.000	.000	.000
649	648.000	6.470	.000	.000	.000	.000	.000
650	649.000	6.474	.000	.000	.000	.000	.000
651	650.000	6.478	.000	.000	.000	.000	.000
652	651.000	6.483	.000	.000	.000	.000	.000
653	652.000	6.487	.000	.000	.000	.000	.000
654	653.000	6.491	.000	.000	.000	.000	.000
655	654.000	6.496	.000	.000	.000	.000	.000
656	655.000	6.500	.000	.000	.000	.000	.000
657	656.000	6.504	.000	.000	.000	.000	.000
658	657.000	6.509	.000	.000	.000	.000	.000
659	658.000	6.513	.000	.000	.000	.000	.000
660	659.000	6.517	.000	.000	.000	.000	.000
661	660.000	6.522	.000	.000	.000	.000	.000
662	661.000	6.526	.000	.000	.000	.000	.000
663	662.000	6.530	.000	.000	.000	.000	.000
664	663.000	6.535	.000	.000	.000	.000	.000
665	664.000	6.539	.000	.000	.000	.000	.000
666	665.000	6.543	.000	.000	.000	.000	.000
667	666.000	6.548	.000	.000	.000	.000	.000
668	667.000	6.552	.000	.000	.000	.000	.000
669	668.000	6.557	.000	.000	.000	.000	.000
670	669.000	6.561	.000	.000	.000	.000	.000
671	670.000	6.565	.000	.000	.000	.000	.000
672	671.000	6.570	.000	.000	.000	.000	.000
673	672.000	6.574	.000	.000	.000	.000	.000
674	673.000	6.578	.000	.000	.000	.000	.000
675	674.000	6.583	.000	.000	.000	.000	.000
676	675.000	6.587	.000	.000	.000	.000	.000
677	676.000	6.591	.000	.000	.000	.000	.000
678	677.000	6.596	.000	.000	.000	.000	.000
679	678.000	6.600	.000	.000	.000	.000	.000
680	679.000	6.604	.000	.000	.000	.000	.000
681	680.000	6.609	.000	.000	.000	.000	.000
682	681.000	6.613	.000	.000	.000	.000	.000
683	682.000	6.617	.000	.000	.000	.000	.000
684	683.000	6.622	.000	.000	.000	.000	.000
685	684.000	6.626	.000	.000	.000	.000	.000
686	685.000	6.630	.000	.000	.000	.000	.000
687	686.000	6.635	.000	.000	.000	.000	.000
688	687.000	6.639	.000	.000	.000	.000	.000
689	688.000	6.643	.000	.000	.000	.000	.000
690	689.000	6.648	.000	.000	.000	.000	.000
691	690.000	6.652	.000	.000	.000	.000	.000
692	691.000	6.657	.000	.000	.000	.000	.000
693	692.000	6.661	.000	.000	.000	.000	.000
694	693.000	6.665	.000	.000	.000	.000	.000
695	694.000	6.670	.000	000	.000	.000	.000
696	695.000	6.674	.000	.000	.000	.000	.000
697	696.000	6.678	.000	.000	.000	.000	.000
698	697.000	6.683	.000	.000	.000	.000	.000
699	698.000	6.687	.000	.000	.000	.000	.000
700	699.000	6.691	.000	.000	.000	.000	.000
701	700.000	6.696	.000	.000	.000	.000	.000

Figure A5. (Sheet 12 of 15)

702	701.000	6.700	.000	.000	.000	.000	.000
703	702.000	6.704	.000	.000	.000	.000	.000
704	703.000	6.709	.000	.000	.000	.000	.000
705	704.000	6.713	.000	.000	.000	.000	.000
706	705.000	6.717	.000	.000	.000	.000	.000
707	706.000	6.722	.000	.000	.000	.000	.000
708	707.000	6.726	.000	.000	.000	.000	.000
709	708.000	6.730	.000	.000	.000	.000	.000
710	709.000	6.735	.000	.000	.000	.000	.000
711	710.000	6.739	.000	.000	.000	.000	.000
712	711.000	6.743	.000	.000	.000	.000	.000
713	712.000	6.748	.000	.000	.000	.000	.000
714	713.000	6.752	.000	.000	.000	.000	.000
715	714.000	6.757	.000	.000	.000	.000	.000
716	715.000	6.761	.000	.000	.000	.000	.000
717	716.000	6.765	.000	.000	.000	.000	.000
718	717.000	6.770	.000	.000	.000	.000	.000
719	718.000	6.774	.000	.000	.000	.000	.000
720	719.000	6.778	.000	.000	.000	.000	.000
721	720.000	6.783	.000	.000	.000	.000	.000
722	721.000	6.787	.000	.000	.000	.000	.000
723	722.000	6.791	.000	.000	.000	.000	.000
724	723.000	6.796	.000	.000	.000	.000	.000
725	724.000	6.800	.000	.000	.000	.000	.000
726	725.000	6.804	.000	.000	.000	.000	.000
727	726.000	6.809	.000	.000	.000	.000	.000
728	727.000	6.813	.000	.000	.000	.000	.000
729	728.000	6.817	.000	.000	.000	.000	.000
730	729.000	6.822	.000	.000	.000	.000	.000
731	730.000	6.826	.000	.000	.000	.000	.000
732	731.000	6.830	.000	.000	.000	.000	.000
733	732.000	6.835	.000	.000	.000	.000	.000
734	733.000	6.839	.000	.000	.000	.000	.000
735	734.000	6.843	.000	.000	.000	.000	.000
736	735.000	6.848	.000	.000	.000	.000	.000
737	736.000	6.852	.000	.000	.000	.000	.000
738	737.000	6.857	.000	.000	.000	.000	.000
739	738.000	6.861	.000	.000	.000	.000	.000
740	739.000	6.865	.000	.000	.000	.000	.000
741	740.000	6.870	.000	.000	.000	.000	.000
742	741.000	6.874	.000	.000	.000	.000	.000
743	742.000	6.878	.000	.000	.000	.000	.000
744	743.000	6.883	.000	.000	.000	.000	.000
745	744.000	6.887	.000	.000	.000	.000	.000
746	745.000	6.891	.000	.000	.000	.000	.000
747	746.000	6.896	.000	.000	.000	.000	.000
748	747.000	6.900	.000	.000	.000	.000	.000
749	748.000	6.904	.000	.000	.000	.000	.000
750	749.000	6.909	.000	.000	.000	.000	.000
751	750.000	6.913	.000	.000	.000	.000	.000
752	751.000	6.917	.000	.000	.000	.000	.000
753	752.000	6.922	.000	.000	.000	.000	.000
754	753.000	6.926	.000	.000	.000	.000	.000
755	754.000	6.930	.000	.000	.000	.000	.000
756	755.000	6.935	.000	.000	.000	.000	.000
757	756.000	6.939	.000	.000	.000	.000	.000
758	757.000	6.943	.000	.000	.000	.000	.000
759	758.000	6.948	.000	.000	.000	.000	.000
760	759.000	6.952	.000	.000	.000	.000	.000
761	760.000	6.957	.000	.000	.000	.000	.000

Figure A5. (Sheet 13 of 15)

762	761.000	6.961	.000	.000	.000	.000	.000
763	762.000	6.965	.000	.000	.000	.000	.000
764	763.000	6.970	.000	.000	.000	.000	.000
765	764.000	6.974	.000	.000	.000	.000	.000
766	765.000	6.978	.000	.000	.000	.000	.000
767	766.000	6.983	.000	.000	.000	.000	.000
768	767.000	6.987	.000	.000	.000	.000	.000
769	768.000	6.991	.000	.000	.000	.000	.000
770	769.000	6.996	.000	.000	.000	.000	.000
771	770.000	7.000	.000	.000	.000	.000	.000
772	771.000	7.004	.000	.000	.000	.000	.000
773	772.000	7.009	.000	.000	.000	.000	.000
774	773.000	7.013	.000	.000	.000	.000	.000
775	774.000	7.017	.000	.000	.000	.000	.000
776	775.000	7.022	.000	.000	.000	.000	.000
777	776.000	7.026	.000	.000	.000	.000	.000
778	777.000	7.030	.000	.000	.000	.000	.000
779	778.000	7.035	.000	.000	.000	.000	.000
780	779.000	7.039	.000	.000	.000	.000	.000

I	DYEQ(I)	VGENV(I)	VUSED(I)	PSV(I)
1	100.000	324.335	28.362	.541
2	105.000	299.957	29.699	.500
3	110.000	276.084	32.269	.460
4	115.000	252.851	36.341	.421
5	120.000	230.273	42.157	.384
6	125.000	208.366	49.950	.347
7	130.000	187.145	60.465	.312
8	135.000	166.631	75.781	.278
9	140.000	146.844	182.815	.245
10	137.500	156.686	87.965	.261
11	138.750	151.778	101.170	.253
12	140.000	146.844	182.830	.245
13	139.375	149.307	124.819	.249
14	139.688	148.112	146.663	.247
15	140.000	146.844	182.835	.245
16	139.844	147.511	163.600	.246
17	139.688	148.112	146.667	.247
18	139.766	147.812	154.885	.246
19	139.727	147.962	150.690	.247
20	139.688	148.112	146.666	.247
21	139.707	148.037	148.642	.247
22	139.697	148.075	147.633	.247
23	139.702	148.056	148.138	.247
24	139.700	148.065	147.886	.247
25	139.701	148.061	148.010	.247
26	139.702	148.056	148.138	.247
27	139.702	148.058	148.072	.247
28	139.701	148.061	148.010	.247
29	139.701	148.059	148.044	.247
30	139.702	148.058	148.072	.247
31	139.701	148.059	148.057	.247
32	139.701	148.058	148.067	.247
33	139.701	148.059	148.062	.247
34	139.701	148.059	148.057	.247
35	139.701	148.059	148.058	.247
36	139.701	148.059	148.062	.247
37	139.701	148.059	148.058	.247

Figure A5. (Sheet 14 of 15)

38	139.701	148.059	148.058	.247
39	139.701	148.059	148.058	.247
40	139.701	148.059	148.058	.247

SOLUTION REACHED, NON-INTERSECTING PROFILES

Figure A5. (Sheet 15 of 15)

Appendix B
Detailed Description of Program EQPR.FOR and Input and Output Files

Introduction

The FORTRAN program EQPR.FOR accepts one input file (EQPR.INP) specifying the characteristics of the original profile and the essential characteristics of the nourishment material and placement geometry. The program also generates *one output file:* EQPR.OUT. The program and the input and output files are described in this appendix. A program listing (EQPR.FOR), and annotated input and output files for Examples 1 and 4 have been presented in Appendix A. In the following presentation, it is necessary to recall that depths and elevations represent positive and negative values of depth h, respectively.

Description of FORTRAN Program and Input Files

This discussion of the FORTRAN program will be assisted by referring to Part 2 and Figure 10 of the main body of this report, the latter of which is also included in this appendix as Figure B1 for reference.

Main program EQPR.FOR

The primary function of the main program is to call the various subroutines that will be described in subsequent sections of this appendix. In addition, in the iterations phase of this program, improved estimates of the equilibrium berm displacement, DYEQ, are generated. The overall sequence of the main program is as follows.

a. Read and write a run description in FORMAT (20A4)

b. Call Subroutine PROCHAR which reads in the initial and placed profile characteristics. Solve, by iteration, for Δy_o, the initial berm displacement. The initial profile can be idealized (of the following form)

$$y = \frac{h}{s_o} + \frac{h^{\frac{3}{2}}}{A^{\frac{3}{12}}}$$

or may be user-specified.

c. Call Subroutine SEDCHAR, which reads in the sediment size information in the form of an idealized (log-normal) distribution or a user-specified distribution.

d. Call Subroutine ATRANS, which converts the sediment size distribution information to a distribution of the sediment scale parameter, *A*. This program reads in the desired number of iterations, ITSMAX, and an initial estimate of the equilibrium profile displacement, DYEQ1.

e. Call Subroutine PSSR, which manages the iteration process, as will be described when discussing this subroutine.

g. The main program develops improved estimates of equilibrium profile displacement DYEQ.

h. Finally, the main program generates output to FILE EQPR.OUT, which summarizes the results of the run.

The paragraphs below describe each of the subroutines.

Subroutine PROCHAR. This subroutine reads in the characteristics of the initial and placed profiles. The first line of input includes the following variables in FORMAT (2I6,8F8.3): JPTYPE, IMAX, AO, SO, SP, VADD, SEQ, HBERM, HSTAR, HMIX.

JPTYPE Type of profile input. If JPTYPE = 0, the depths are calculated as an equilibrium profile using Equation 4. If JPTYPE = 1, the depths are read in as individual points at arbitrary spacings across the profile.

IMAX =	Number of final points across the profile.
AO =	Profile scale parameter of initial profile (only required if JPTYPE = 0).
DY =	Spacing of points across final profile.
SO =	Beach face slope of initial parameter (only required if JPTYPE = 0).
SP =	Slope of placed material.
VADD =	Volume of added material in cubic meters/meter.

SEQ =	Slope of above water portion of equilibrium profile.
HBERM =	Height of berm in meters.
HSTAR =	Depth of effective motion, i.e., the depth to which the nourishment material is considered to equilibrate.
HMIX =	The depth to which nourished sand is mixed in the placement area.

This completes information contained on the first card. The program then calls Subroutine PROCAL if JPTYPE = 0 to calculate the initial profile. If JPTYPE = 1, the program reads in IMAXP, the number of pairs of points of (YV,HV0) to be provided as input. These values are read in FORMAT (8F8.2). Appendix A has provided annotated input files for Examples 1 and 4, respectively, which are for idealized and user-specified initial profiles. The user-specified profile (YV,HV0) is then interpolated by Subroutine INTERP to establish IMAX pairs of (Y,H0) at spacing DY.

With the information above describing the initial profile and nourishment characteristics, the <u>initial</u> shoreline advancement due to nourishment is calculated. The characteristics of this profile are a berm at height HBERM extending a distance DYO from the initial shoreline, sloping at SP to intersection with the initial profile (H0) and containing VADD volume of sediment. The shoreline advancement DYO is determined by iteration.

This completes description of the Subroutine PROCHAR.

Subroutine SEDCHAR. This subroutine reads in the characteristics of the nourishment material. The first card has the following format (I6,2F8.2):

JSED =	Type of sediment input. If JSED = 0, the sediment characteristics are described by Equation 1 and XMU, the sand size mean in phi units and SIG, the sample standard deviation, also in phi units. If JSED = 1, the sediment characteristics are read in as individual pairs of (diameter, cumulative distribution).
LMAX =	Number of pairs of (sand size, decimal percentage) to be generated (JSED = 0) or read in (JSED = 1) to define the cumulative sand size distribution.
XMU =	Sand size in phi units (JSED = 0).
SIG =	Sand standard deviation in phi units (JSED = 0).

The second through sixth cards read by this subroutine establish a table of 19 pairs of (DI,AI) values which represent the sediment scale parameter variation AI with diameter DI. *These values will be the same for all runs and are simply a means of inputting the empirical information in Figure B2.*

If JSED = 0, the cumulative distribution of the nourishment sediment is calculated in accordance with Equation 1. If JSED = 1, the cumulative distribution is read in as pairs of (D(L), P(L), L = 1, LMAX) in format

(8F8.2) where D(L) is the diameter in millimeters and P(L) is the probability in decimal percentage. Note $P(1) = 0$, and $P(\mathrm{LMAX}) = 1.0$.

The reader is referred to Appendix A for examples of input to SEDCHAR for idealized (Figure A2) and user-specified (Figure A4) sediment characteristics, respectively. This completes description of the Subroutine SEDCHAR.

Subroutine ATRANS. This subroutine simply calculates sediment scale values A for each of the diameters D in the probability distribution. Thus the distribution of $P = f(D)$ is transformed to $A = f(P)$, where P is the decimal percentage coarser than.

At this stage, the initial profile, placed profile, and sediment characteristics (including their A values) are established. The main program then reads in the number of iterations to be performed, ITSMAX, and an initial estimate for the equilibrium shoreline advancement, DYEQ (the estimate is termed DYEQ1).

The main program calls Subroutine PSSR, which carries out the solution.

Subroutine PSSR. This subroutine establishes the equilibrium profile locally consistent with the nourishment volume mobilized and grain size distribution. In this process, the program considers three regions as discussed below and as illustrated in Figure B2.

Iterations are carried out until the volume of sand mobilized is equal to that deposited out to the point where the equilibrium and initial profiles intersect (Intersecting Profiles) or the equilibrium profile reaches HSTAR (Non-intersecting Profiles). Common to all regions is that the equilibrium profile is advanced seaward by increments of DY and at each advancement, the volume generated from the placement area, VGEN, is calculated (if appropriate) and the volume used (VUSED) is also calculated. Also in advancing seaward, the equilibrium profile computations consider coarsest sand to be deposited in the more landward portions of the profile and, as the profile advances still farther seaward, finer and finer sediment is utilized.

a. Region I - This is the region in which sediment is removed from the *placement* volume. Above the water, the slope is considered uniform as SEQ. Throughout this region, the volume generated is considered to include, in addition to the net volume removed from the placed geometry, a depth HMIX, which is user-specified. The equilibrium profile is extended to intersection with the *placed* profile. All of the material removed up to this point (including the contribution of HMIX) represents VGEN. The only volume used (VUSED) up to this point is that due to HMIX. Seaward of this intersection point, VGEN does not change and VUSED increases as the summation of the

elemental contributions between the equilibrium profile and the underlying profile, which could be the placed or initial profile.

b. Region II - This region lies between the intersection points of the equilibrium and placed profiles and the initial and placed profiles.

c. Region III - This region extends from the intersection of the initial and placed profile to the end of the equilibrium profile, which, as noted previously, can occur due to profile intersection or the equilibrium profile reaching the depth of effective motion.

In the calculations of the equilibrium profile, the Subroutine PSSR calls two Subroutines, YNEW and VOL, each of which is described below.

a. Subroutine YNEW - This subroutine advances the equilibrium profile by a distance DY. In carrying out the calculations, an estimate of the current *A* value is necessary. This is determined from an estimate of the *total* value of VGEN for the profile and the current value of VUSED. The *A* value is then determined from A=f(P) where P = VUSED/VGEN. This interpretation is accomplished by Subroutine INTERP.

b. Subroutine VOL - This subroutine calculates, depending on the region of equilibrium profile (vis-a-vis, the initial and placed profiles), the volume increments to VGEN and VUSED for each increment of equilibrium profile advancement. In this process, it is extremely important that the details of the volumetric contribution be evaluated correctly at the intersection points (equilibrium and placed; placed and initial). This subroutine calls Subroutine CROSS, which establishes the distance YINT and depth HCROSS of the intersection point of the placed and equilibrium profiles.

The output file EQPR.OUT information is reasonably well defined by the output annotation. There are six different types of information presented. The first line is simply a repeat of the run descriptor, which is read in as the first card in EQPR.INP. Next are the input pairs of DI, AI also provided in the input files to all runs. Thirdly, the generated or input pairs of sediment size and cumulative probability (D(I), P(I)) are presented. The next, and by far the largest table, includes the variation of the following with distance, Y: HO(I), HP(I), HEQ(I), AC(I), VUSED(I), and PCC(I), where all terms have been described previously, except for PCC(I), which is the value of the proportion of volume used up to that point relative to the volume available. Note that seaward of the intersection points of the initial (HO(I)) and placed (HP(I)) profiles, the placed profile values are given as zero and similarly for the equilibrium profile seaward of intersection with the initial profile or seaward of reaching HSTAR. The last table presents a summary of the following information for each iteration value: DYEQ(I), VGEN(I), VUSED(I), and PSV(I). Each of these values has been defined before with

the exception of PSV(I), which is the ratio of total volume used (and generated) to volume added. Note that for the solution to be successful, VGEN must equal VUSED. The last line of the output file announces that a solution has been reached and whether the solution is of the intersecting profile or non-intersecting profile type.

This completes the detailed description of the computer program EQPR.FOR, the input file EQPR.INP, and the output file EQPR.OUT. Annotated input and output files for Examples 1 and 4 have been presented in Appendix A.

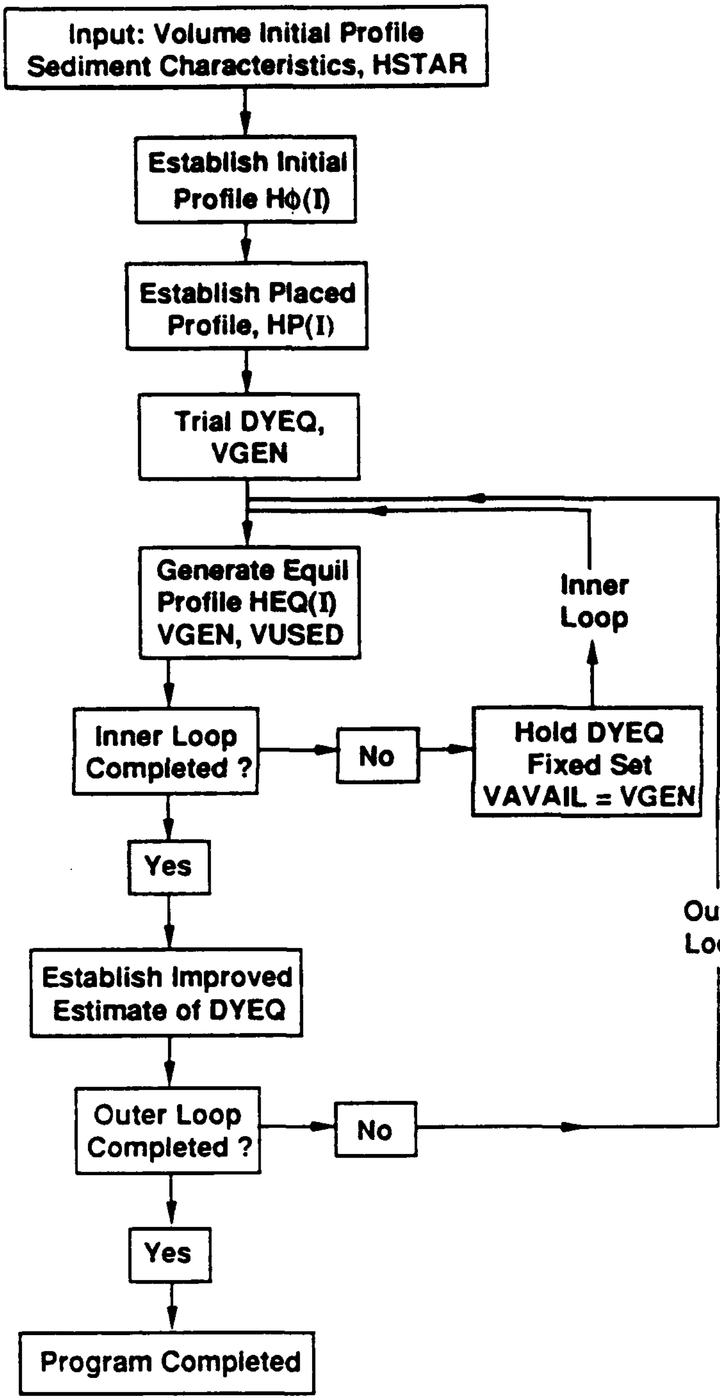

Figure B1. Flow diagram for problem solution

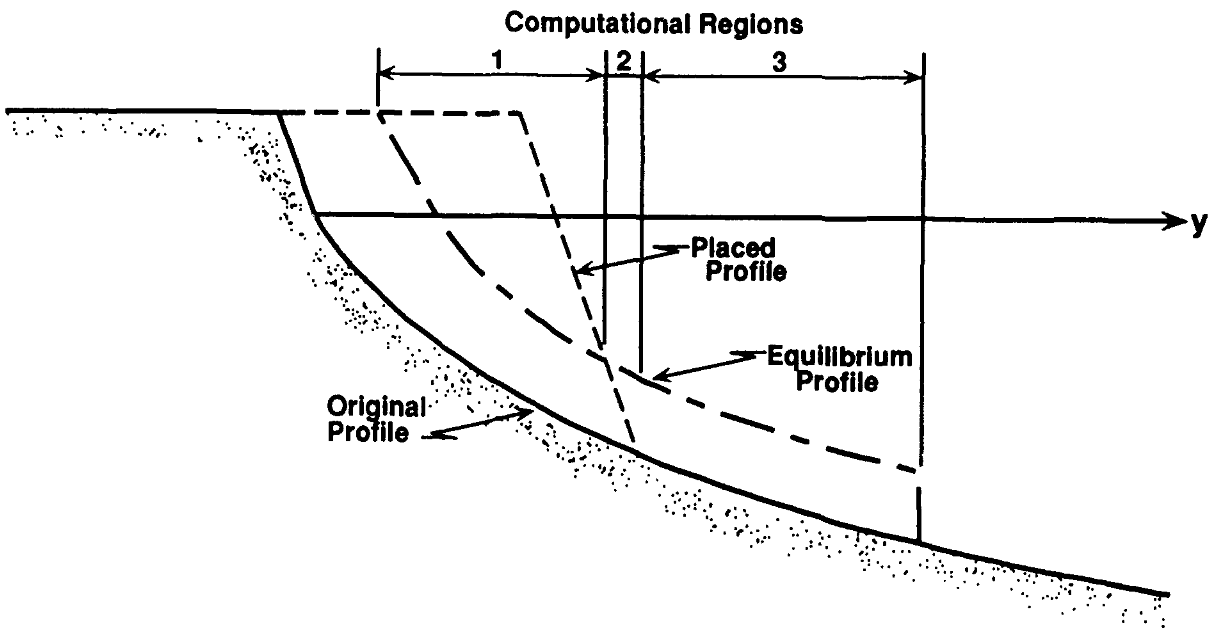

Figure B2. Three regions considered in computational process

Appendix C Additional Data for Delray Beach, Florida

This appendix presents additional beach profile and grain size distribution data for Delray Beach, Florida. Although these data have not been analyzed/evaluated to date, they are included in the report as a potentially valuable data source. The 14 plots are described briefly below.

a. Figure C1 presents the average grain size across Delray Beach in 1976. These results are for a composite sample based on several profiles. Figure C2 presents the grain size distributions associated with each of the four points in Figure C1.

b. Figures C3, C4, C5, and C6 present beach profiles for pre-nourishment (1973), and post-nourishment (1983 and 1988) for Stations R177, R180, R184, and R187, respectively.

c. Figures C7, C8, C9, and C10 present grain size distributions in 1988 for six locations across the profiles at Stations R177, R180, R184, and R187, respectively.

d. Figures C11, C12, C13, and C14 present the 1988 profiles and associated grain size distributions at three locations across these profiles at Stations R177, R180, R184, and R187, respectively.

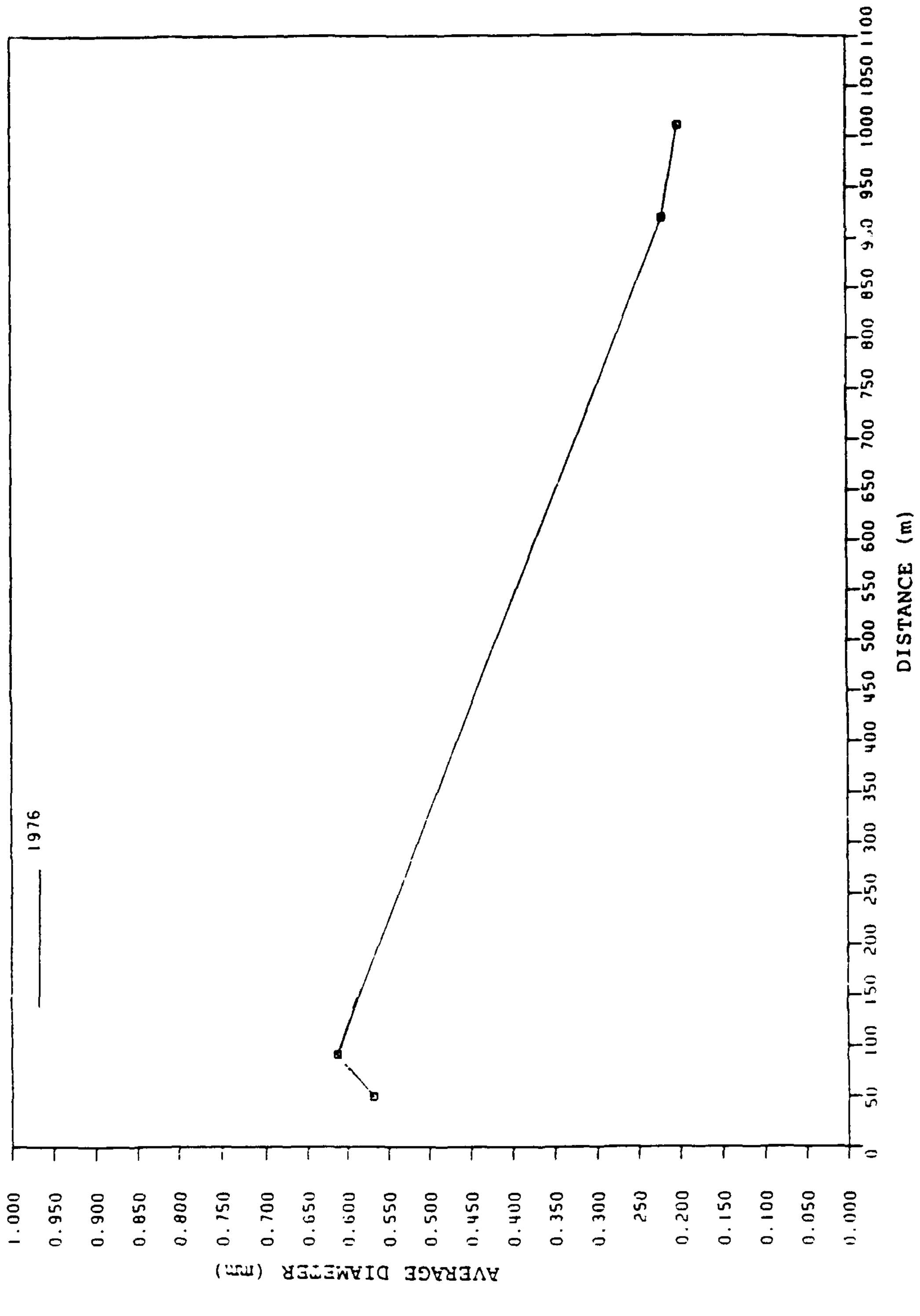

Figure C1. Average grain size distribution across Delray Beach, Florida, 1976

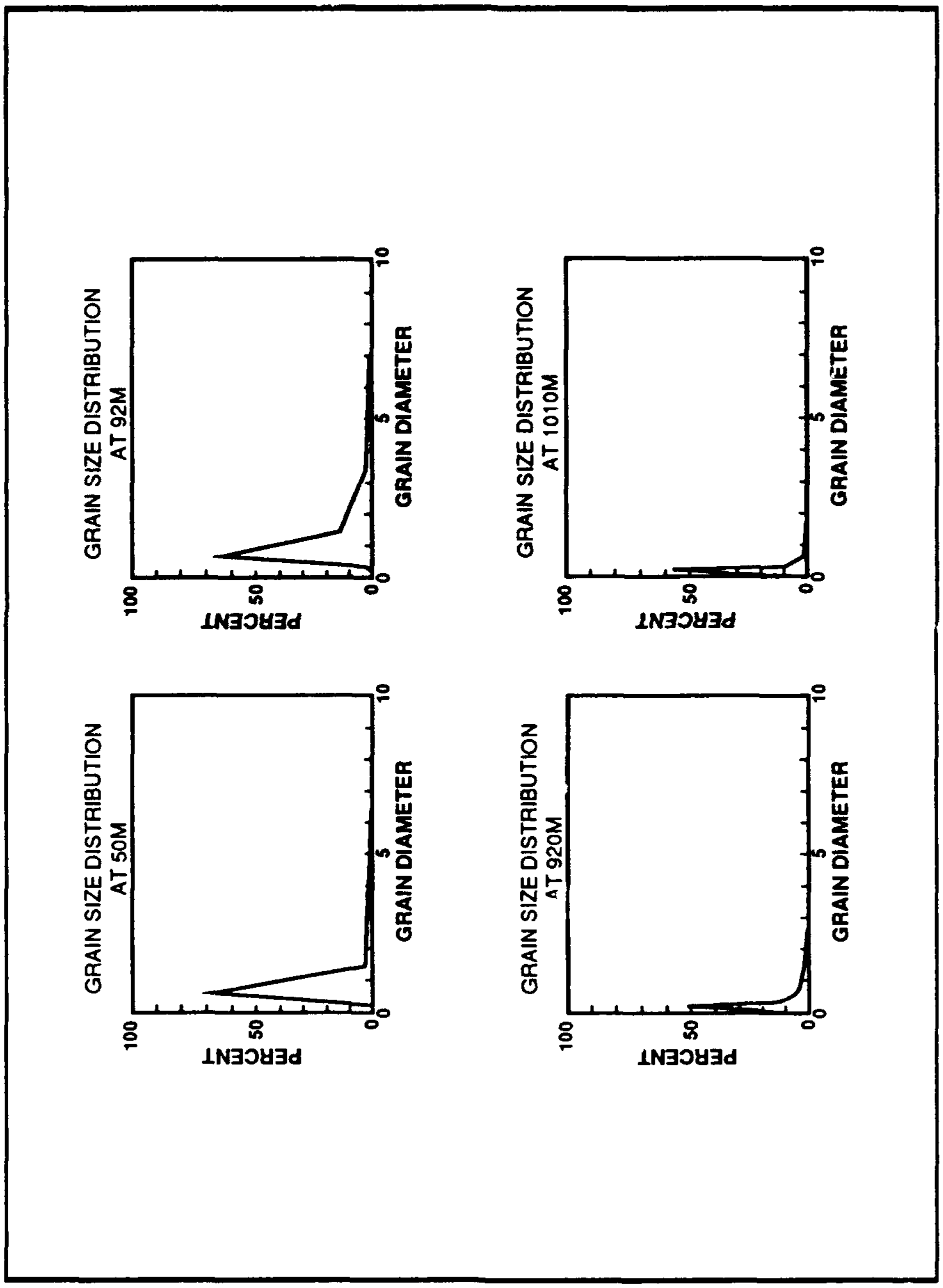

Figure C2. Grain size distributions at various locations across the profile, Delray Beach, Florida, 1976

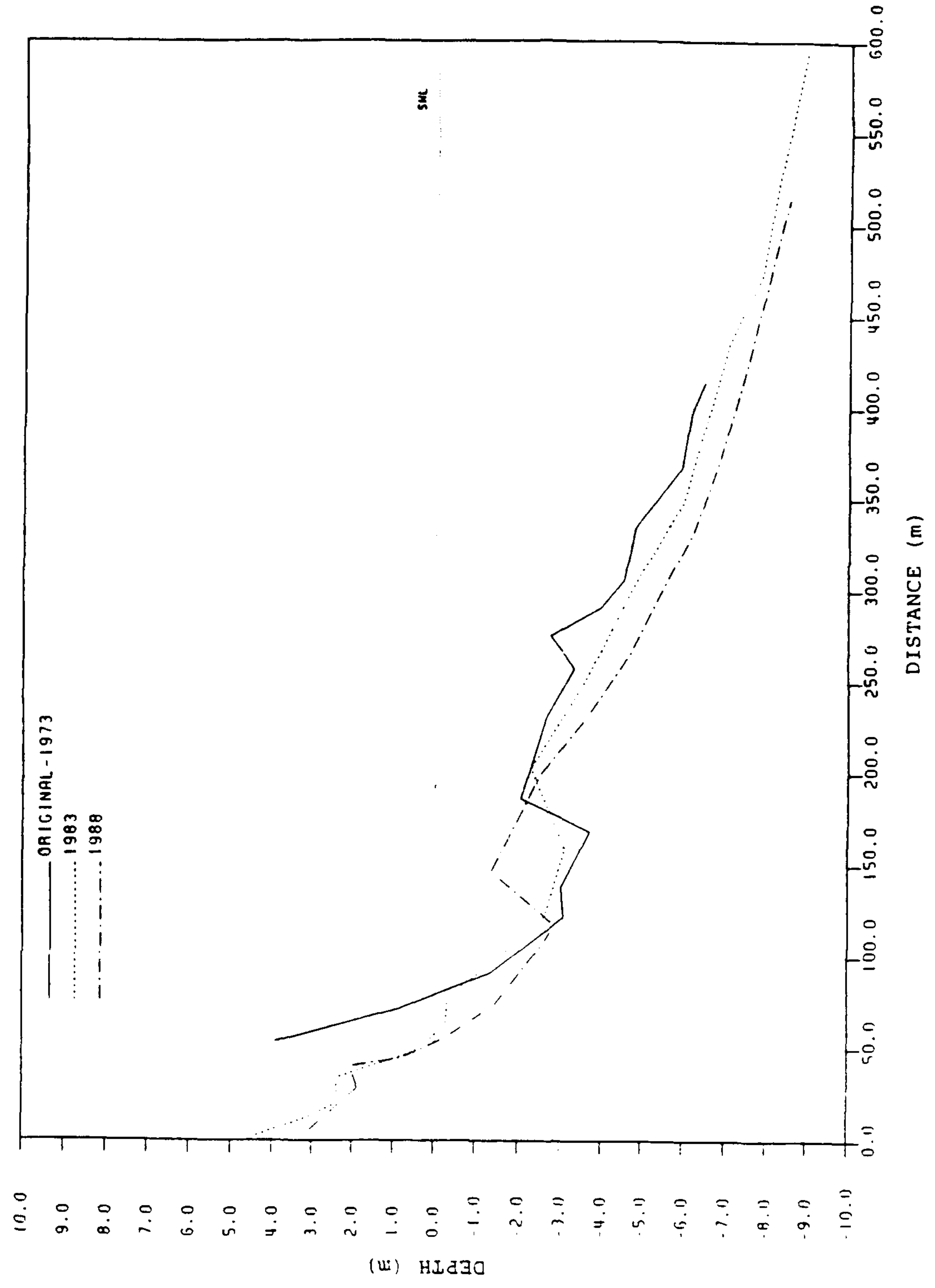

Figure C3. Beach profiles at R177 for 1973, 1983, and 1988, Delray Beach, Florida

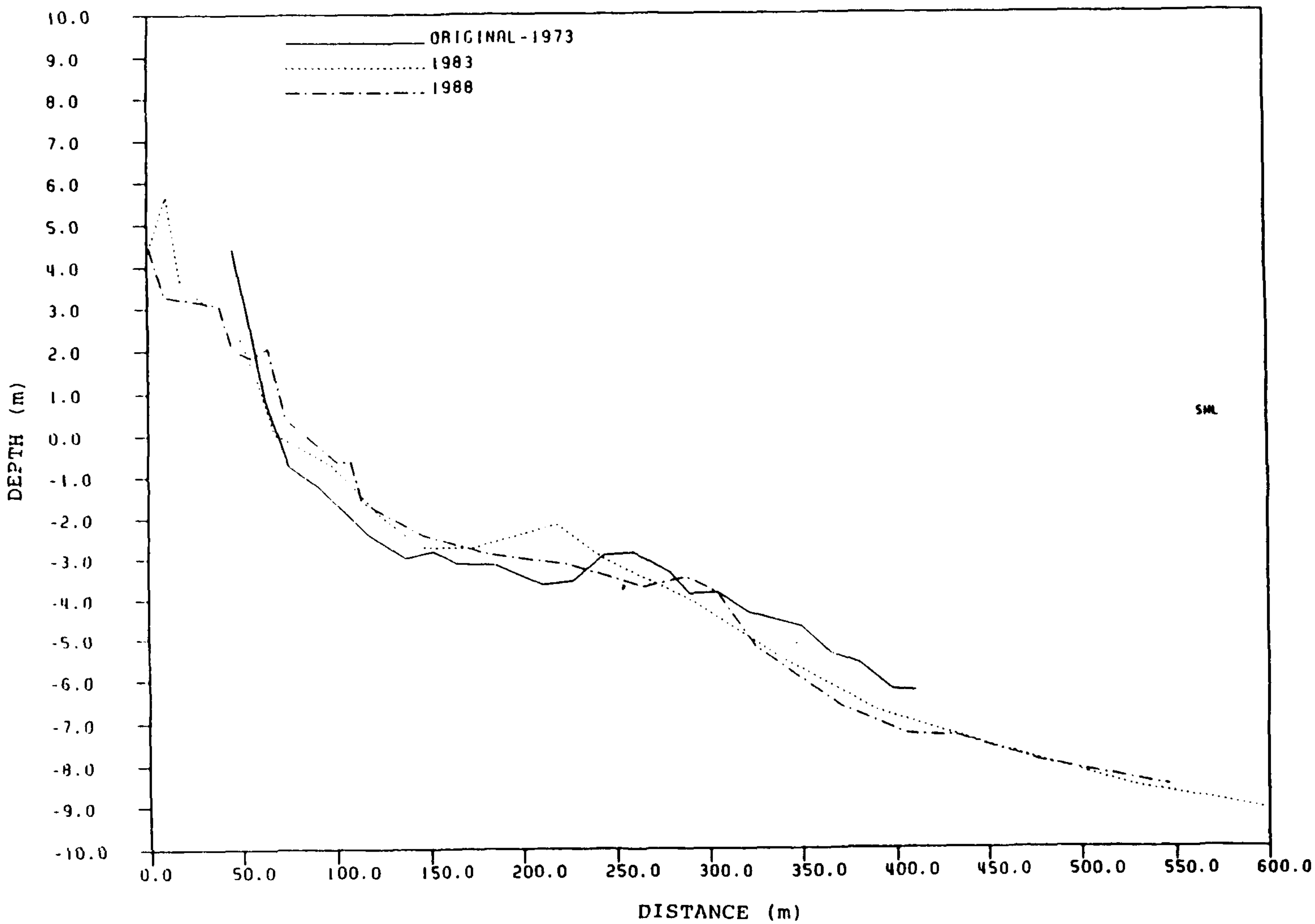

Figure C4. Beach profiles at R180 for 1972, 1983, and 1988, Delray Beach, Florida

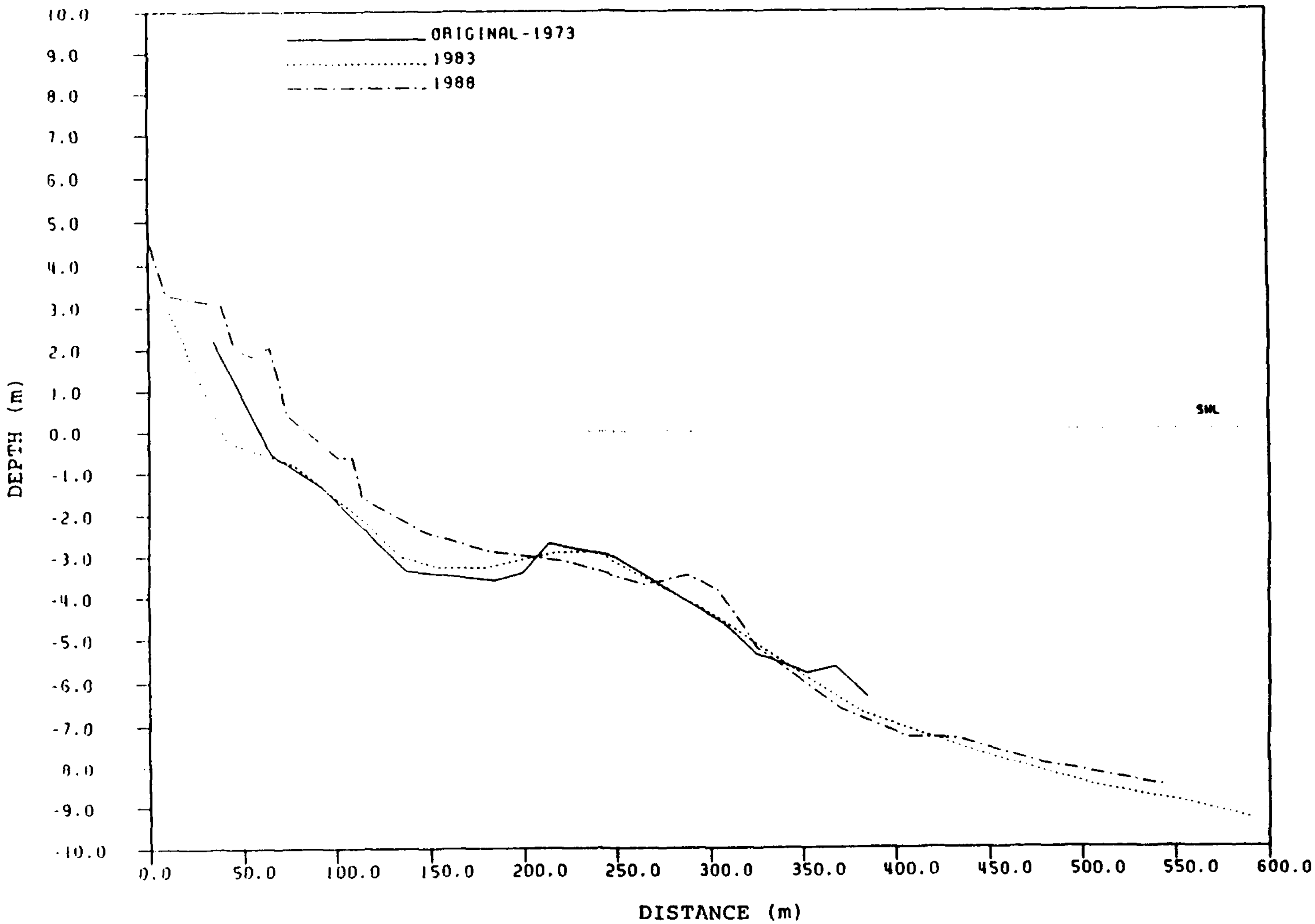

Figure C5. Beach profiles at R184 for 1973, 1983, and 1988, Delray Beach, Florida

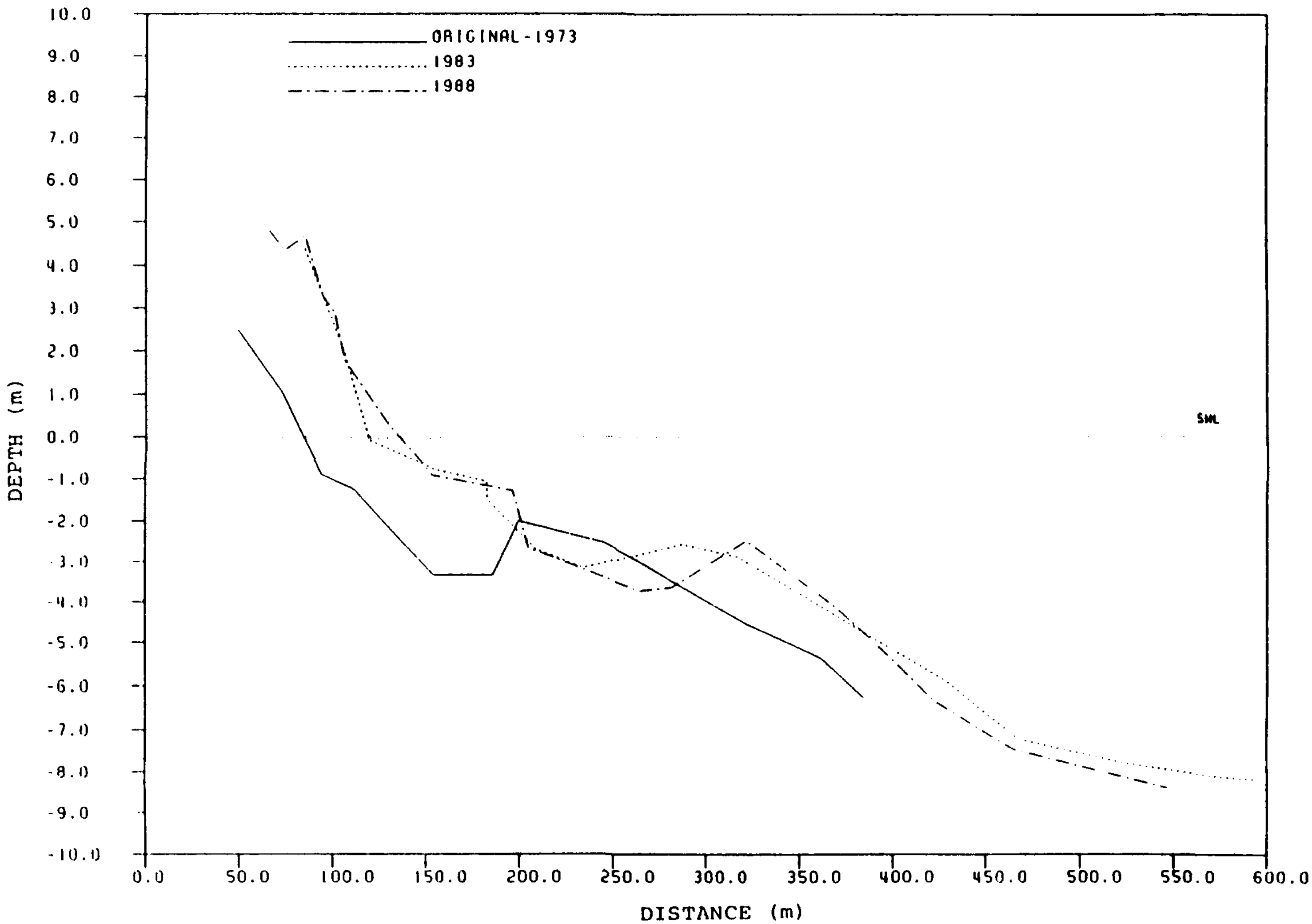

Figure C6. Beach profiles at R187 for 1973, 1983, and 1988, Delray Beach, Florida

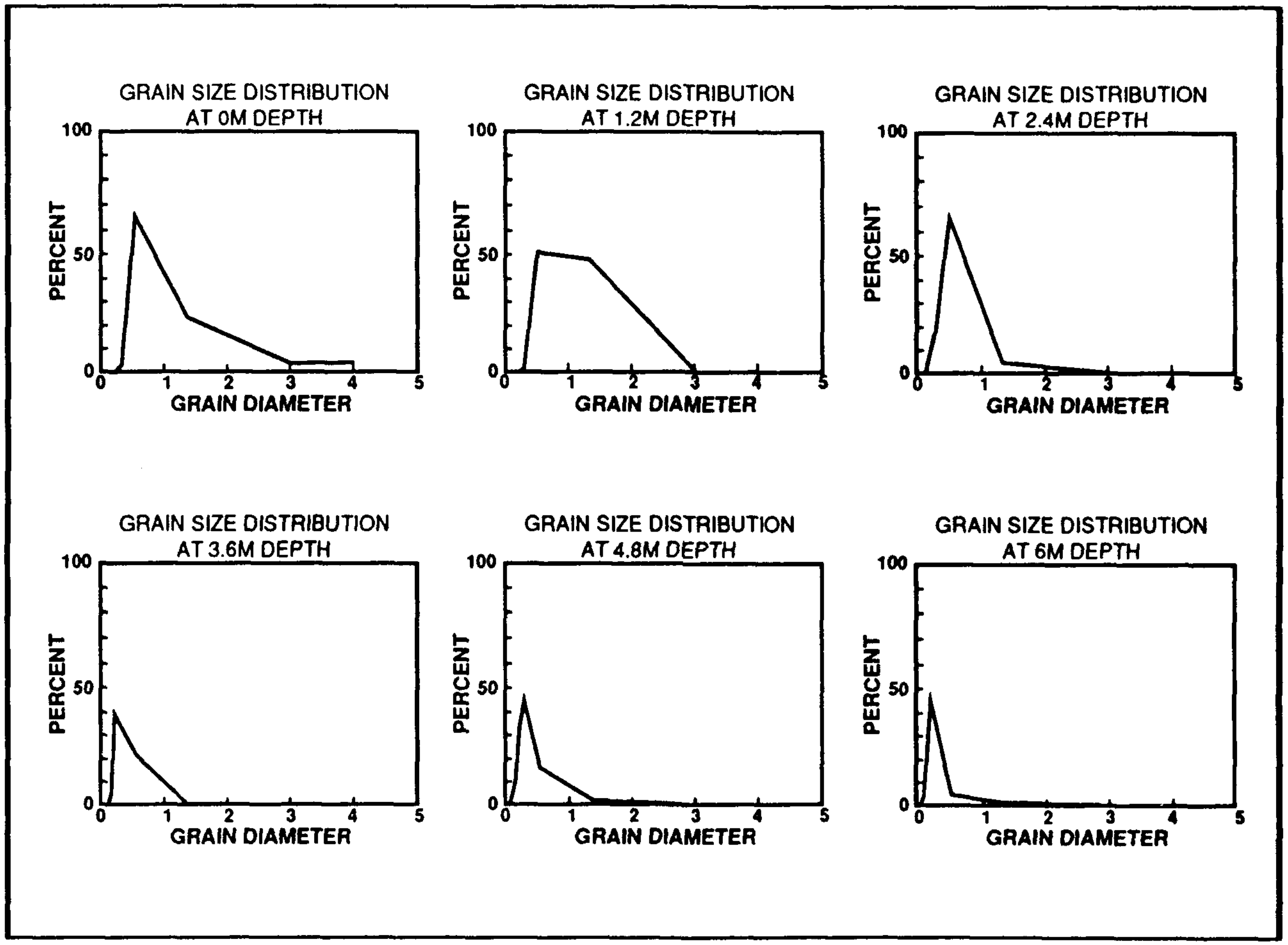

Figure C7. Grain size distributions at various locations across profile R177, Delray Beach, Florida, 1988. Grain diameter in millimeters

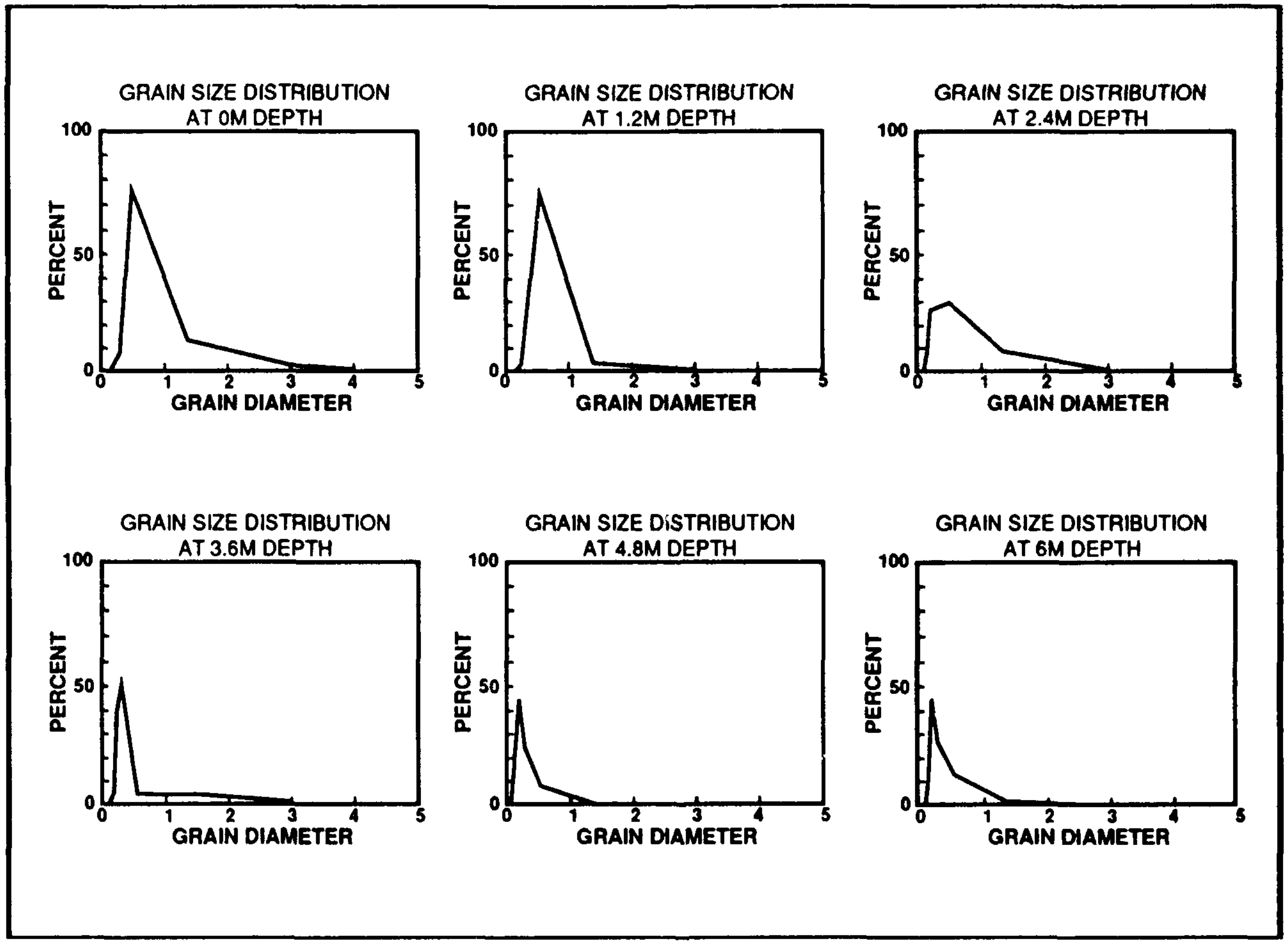

Figure C8. Grain size distributions at various locations across profile R180, Delray Beach, Florida, 1988. Grain diameter in millimeters

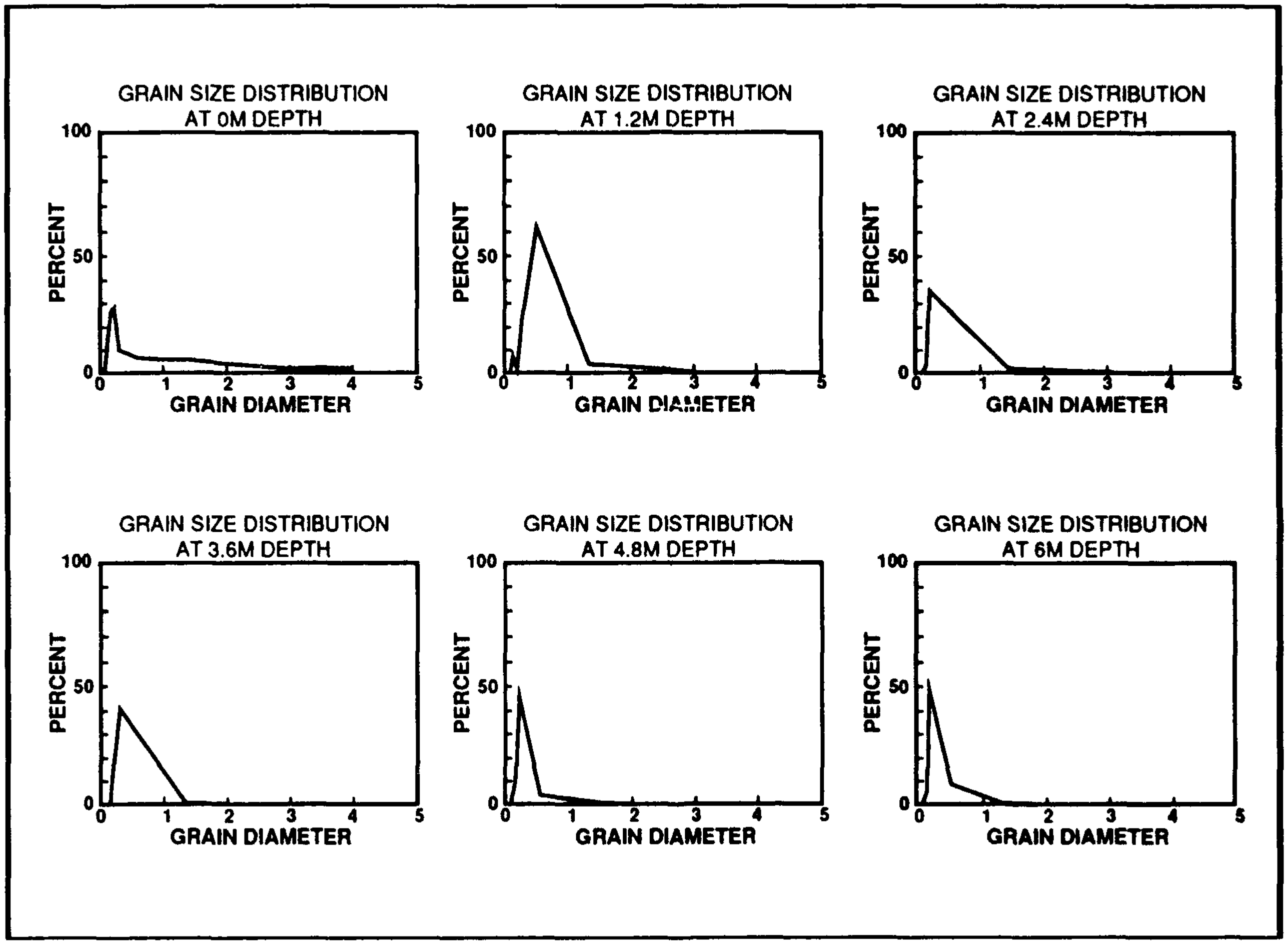

Figure C9. Grain size distributions at various locations across profile R184, Delray Beach, Florida, 1988. Grain diameter in millimeters

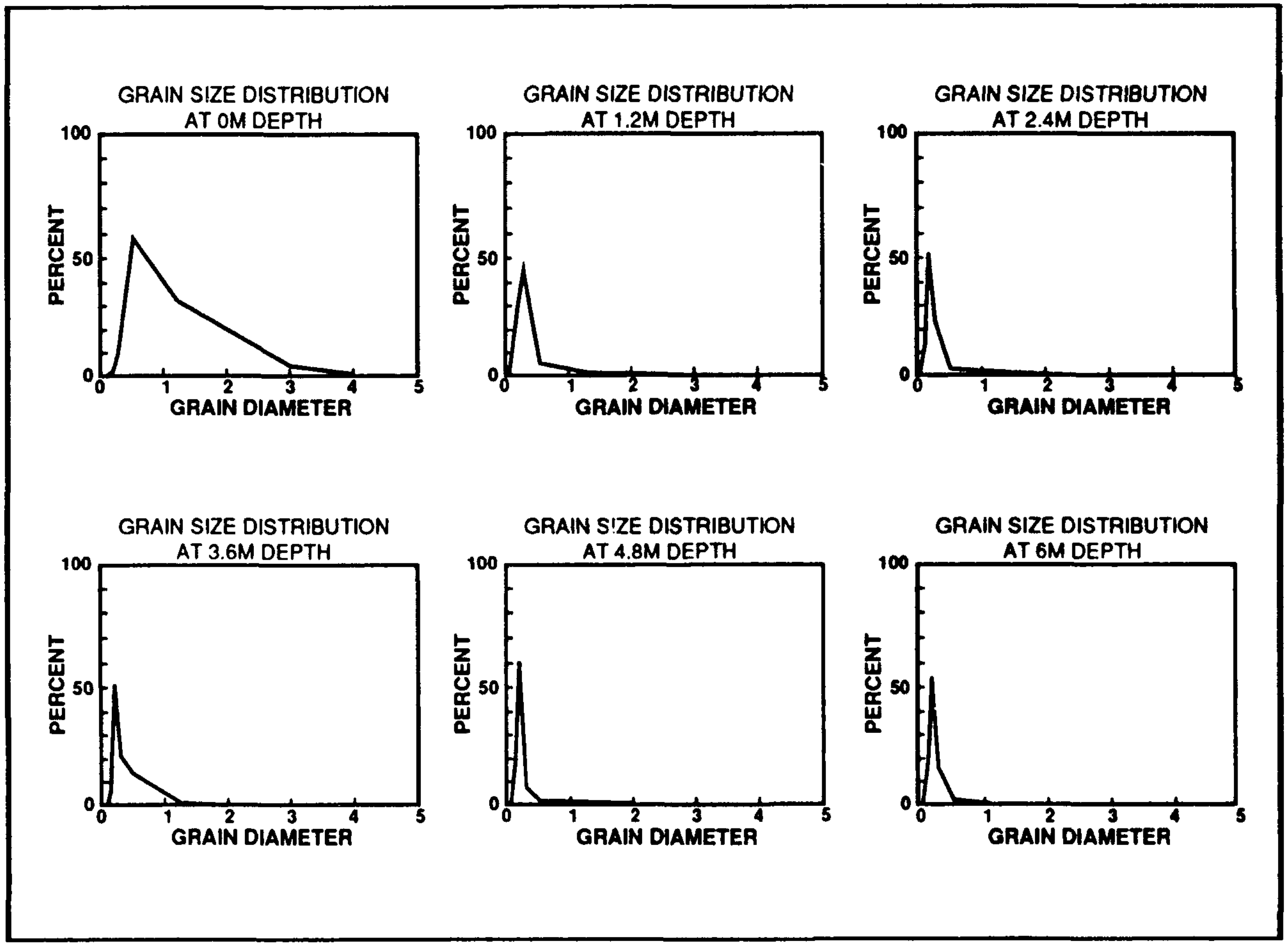

Figure C10. Grain size distribution at various locations across profile R187, Delray Beach, Florida, 1988. Grain diameter in millimeters

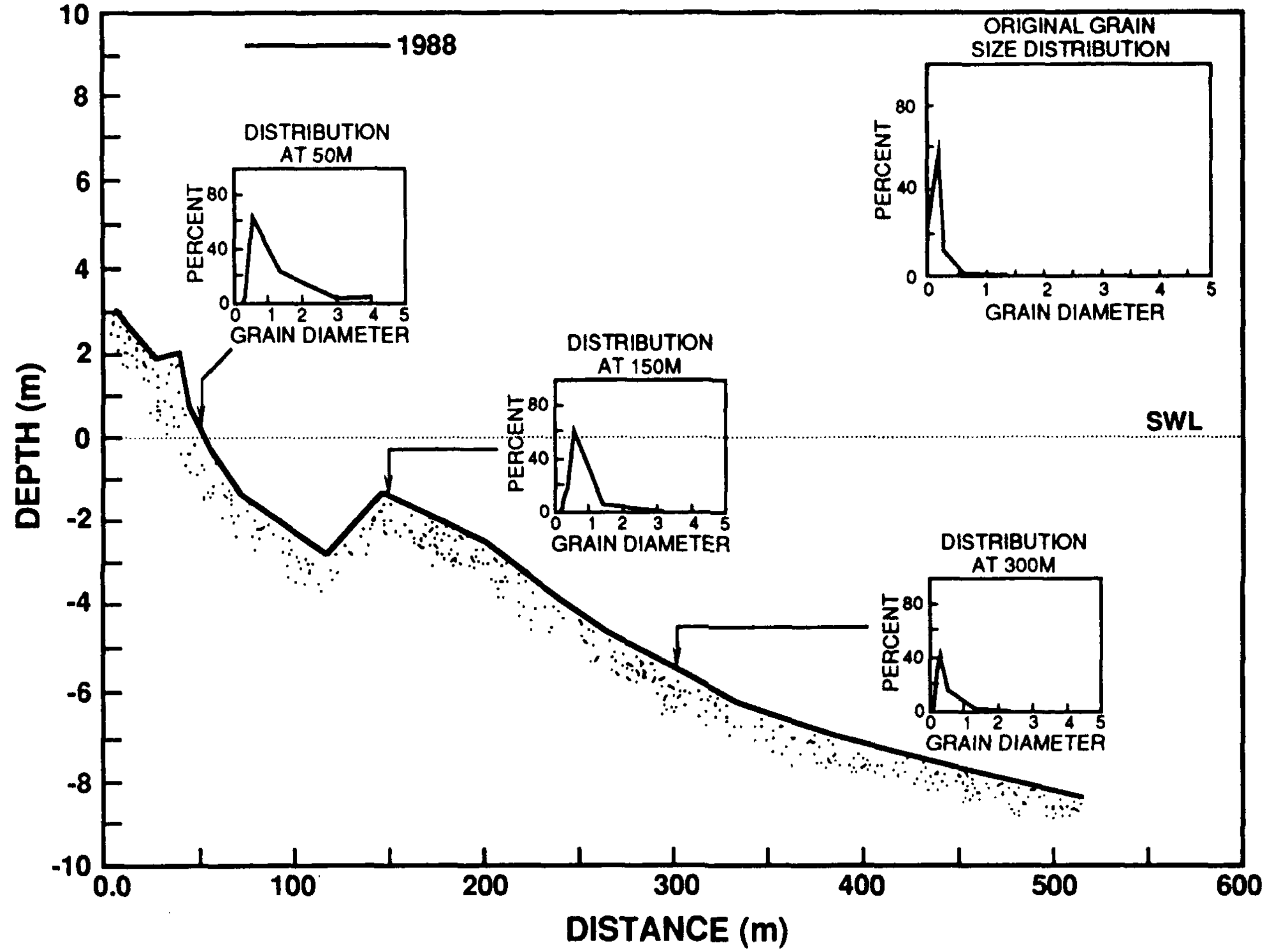

Figure C11. Ceach profile at Station R177 in 1988 at Delray Beach, Florida, and associated grain size distributions. Grain diameter in millimeters

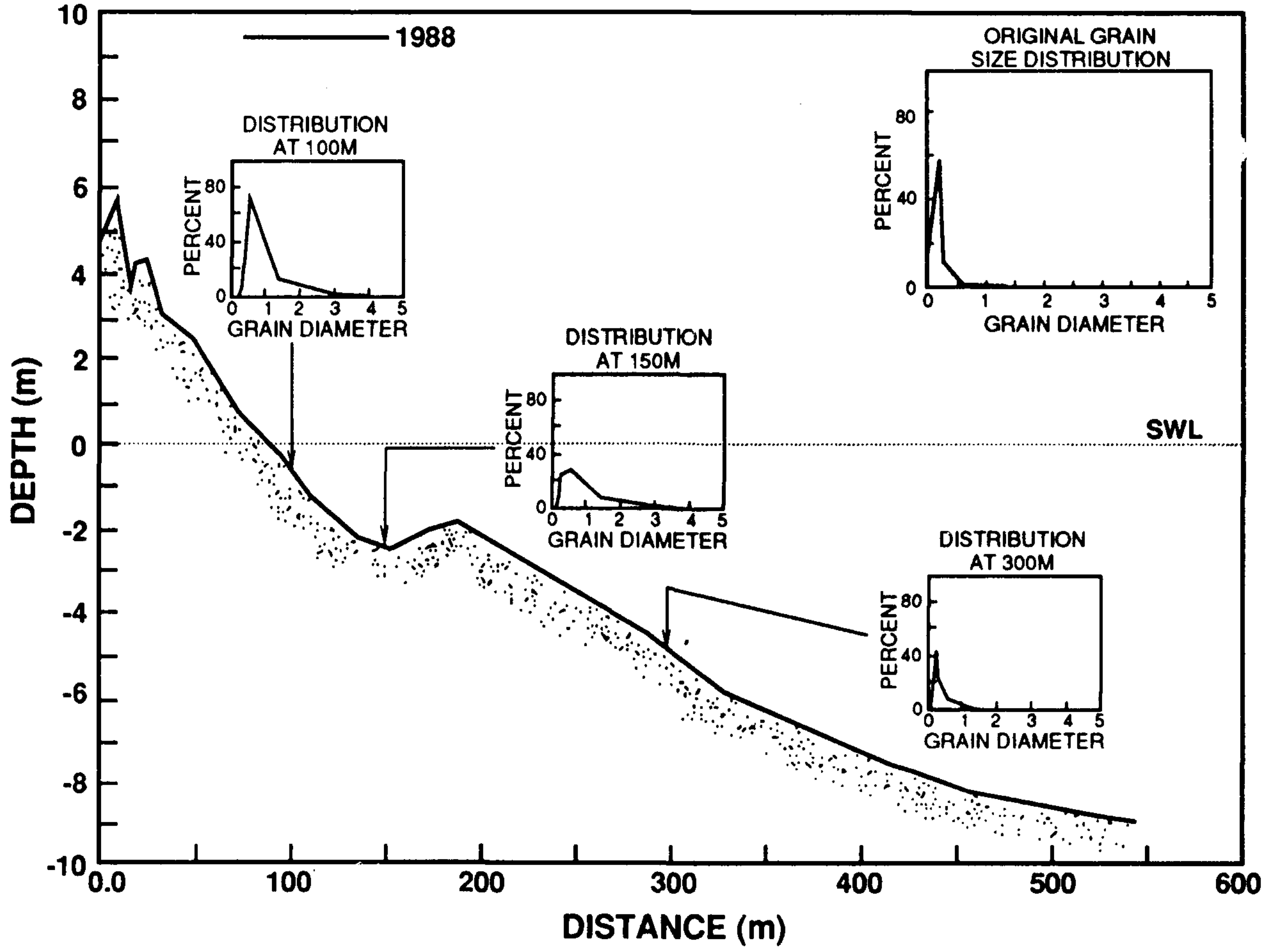

Figure C12. Beach profile at Station R180 in 1980 at Delray Beach, Florida, and associated grain size distributions. Grain diameter in millimeters

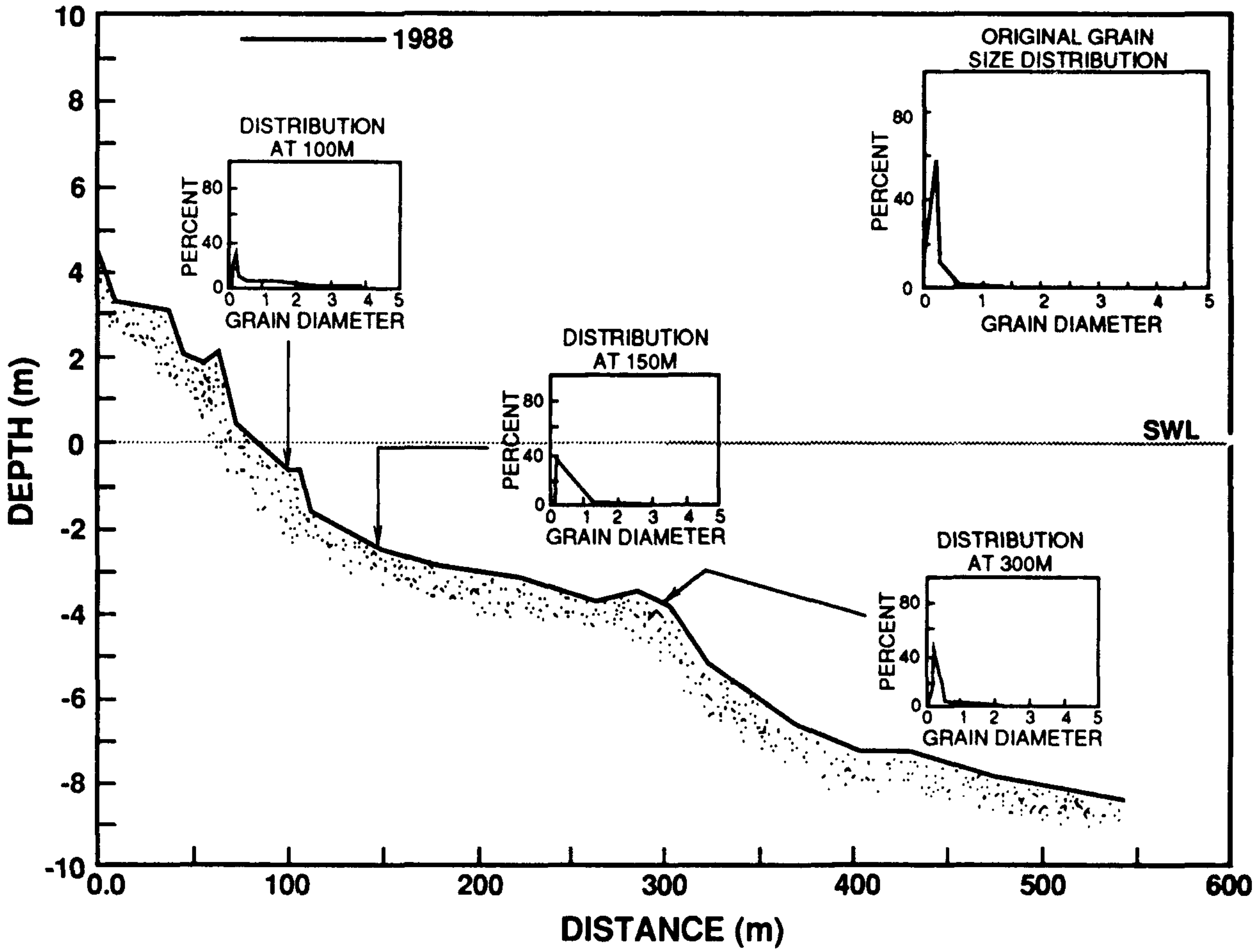

Figure C13. Beach profile at Station R184 in 1988 at Delray Beach, Florida, and associated grain size distributions. Grain diameter in millimeters

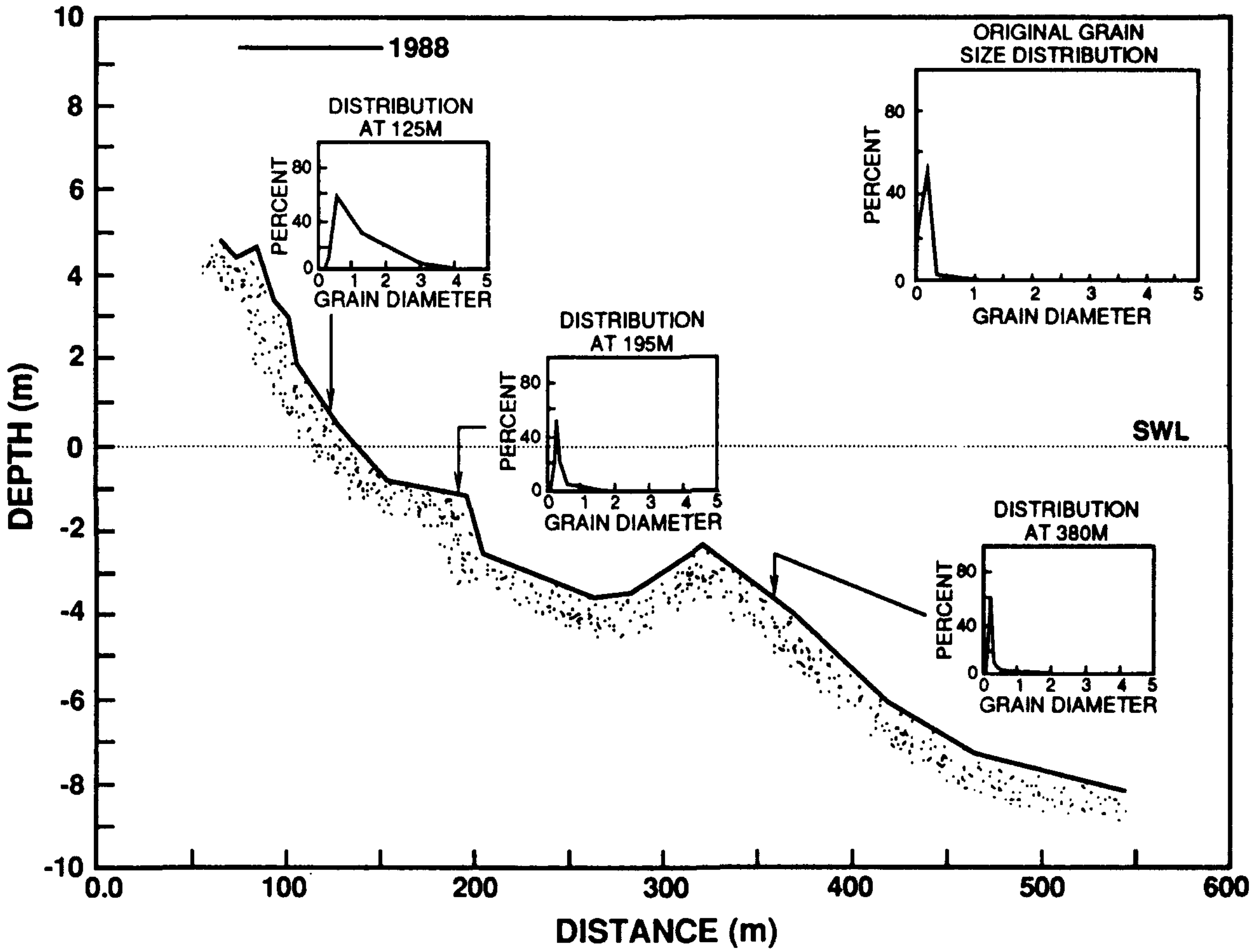

Figure C14. Beach profile at Station R187 in 1988 at Delray Beach, Florida, and associated grain size distributions. Grain diameter in millimeters

Appendix D Notation

A	Sediment scale parameter, $m^{1/3}$
A_F	Nourishment sediment scale parameter, $m^{1/3}$
A_N	Native sediment scale parameter, $m^{1/3}$
B	Berm height, m
D	Sediment grain size, mm
e	Euler's constant (2.71828...)
f	General function; grain size distribution
F	Convergence factor
h	Water depth, m
h_o	Initial profile depth, m
h_p	Placed profile depth, m
h_{EQ}	Equilibrium profile depth, m
h_*	Depth of limiting sediment motion, m
Δh_{mix}	Thickness to which sediment is mixed in placed portion of cross section, m
k	Iteration number
K	Empirical coefficient, m^{-1}
m	Empirical coefficient describing best-fit variation in cross-shore direction
s	Beach slope
s_o	Initial profile slope
s_p	Nourishment placement slope
$\forall$	Nourishment volume placed per unit beach length, m^2

w	Sediment fall velocity, m/sec
W_*	Width of active profile for the initial native profile, m
y	Distance seaward from the shoreline
Δ	Signifies small change in quantity
σ	Sediment diameter in phi units
μ	Sediment sample mean in phi units
$\acute{\sigma}$	Sediment sample deviation in phi units

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13. ABSTRACT *(Maximum 200 words)*

The rational design of beach-nourishment projects requires the ability to calculate the geometry of the added sand volume. This capability is essential for quantitative evaluation of relative merits of various borrow areas and in benefit/cost analysis of such projects. In many cases, the material may be a by-product of a dredging project carried out for purposes other than beach nourishment, and the material dredged or to be placed may have a different grain-size distribution than the original or native beach.

This report presents a new methodology for predicting the equilibrium beach profile resulting from placement of an arbitrary volume of material with an arbitrary grain-size distribution on a profile of arbitrary shape and grain-size distribution. The methodology developed, which depends on the theory of equilibrium profile shape, is proposed as an alternative to traditional compatibility and overfill ratio factors for borrow and native material. The methodology considers two-dimensional (cross-shore) conditions.

The theory is first developed and characteristics of equilibrium beach profiles relevant to beach-nourishment projects are presented. The theory is then applied in four specific examples covering a range of beach, native sediment, and fill material conditions.

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Beach profile change	Equilibrium profile

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